

Statistical Arbitrage with Uncertain Fat Tails

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Abstract

I develop a model of statistical arbitrage trading in an environment with “fat-tailed” information. If risk-neutral arbitrageurs are uncertain about the variance of fat-tail shocks and if they implement max-min robust optimization, they will choose to ignore a wide range of pricing errors. Although model risk hinders their willingness to trade, arbitrageurs can capture the most profitable opportunities because they follow a linear momentum strategy beyond the inaction zone. This is equivalent to a machine-learning algorithm called LASSO. Arbitrageurs can also amass market power due to conservative trading under this strategy. Their uncoordinated exercise of robust control facilitates tacit collusion, protecting their profits from being competed away even if their number goes to infinity. In an extended model where an insider strategically interacts with those arbitrageurs, the insider can induce them to trade too aggressively, giving herself a reversal trading opportunity. Doing so distorts price informativeness and threatens market stability.

Keywords: fat tails, robust control, cartel effect, machine learning, price manipulations.

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1 Introduction

In finance, extreme movements of asset prices occur much more frequently than predicted by the tail probabilities of a Gaussian distribution. Such *fat-tail* events have caused many problems, as exemplified by the failure of *Long Term Capital Management*. It is error-prone to predict fat-tail events or to deal with their higher-order statistics. These difficulties give rise to model risk¹ and drive traders to implement robust control. Model risk is a prominent concern for arbitrageurs whose activities are essential for market efficiency. Little is known about how model risk affects arbitrage trading in a fat-tail environment. This topic is both practically relevant and theoretically challenging. Answers to this question can provide new insights into many topics in asset pricing, risk management, and market regulation.

The existence of various anomalies such as momentum suggests that financial markets are not completely efficient². Statistical arbitrage opportunities are also indicative of price inefficiency, because arbitrageurs can make profits given only public information³. To study statistical arbitrage trading, I introduce random fat-tail shocks to disrupt the efficient market of a two-period Kyle (1985) economy. In the standard Kyle model setup, an informed trader privately observes the stock liquidation value and trades sequentially to maximize her profits, under the camouflage of noise traders and against competitive market makers. A Gaussian information structure permits a unique linear equilibrium with an efficient linear pricing rule.

This paper models the stock value as a random realization drawn from the mixture of Gaussian and Laplacian distributions, which have the same mean and variance. It is only observed by an informed trader. The choice of a Laplacian distribution is empirically well-grounded⁴. It has fat tails on both sides since its probability density decays exponentially. This mixture setup allows the stock value to be *fat-tailed* with some probability. Market makers believe that they live in the Gaussian world and also regard it as the common belief among all agents. Market makers have the correct prior about the mean, variance, and skewness, but incorrect beliefs about higher moments of the stock value distribution. With Gaussian beliefs, they keep using a linear pricing rule⁵, which can result in estimation bias if fat-tail shocks occur. This invites arbitrageurs to correct pricing errors. By assumption, arbitrageurs are sophisticated enough to distinguish the distribution types (i.e., mispricing

¹Model risk is the risk of loss when traders use the wrong model or deal with uncertain model parameters.

²As documented by Jegadeesh and Titman (1993), the momentum strategy could earn abnormal returns.

³See Lehmann (1990), Campbell, Lo, and MacKinlay (1997), Bondarenko (2003), Hogan, Jarrow, Teo, and Warachka (2004), and Gatev, Goetzmann, and Rouwenhorst (2006) for discussions.

⁴The Laplace distribution can well characterize the distributions of stock returns sampled at different time horizons. This is documented, for example, by Silva, Prange, and Yakovenko (2004).

⁵The empirical price impact function, which measures the average price change in response to the size of an incoming order, is roughly linear with slight concavity. See Loeb (1983), Grinold and Kahn (2000) [p. 453], Gabaix, Gopikrishnan, Plerou, and Stanley (2006), and Kyle and Obizhaeva (2016).

cases), but they face uncertainty about the dispersion of Laplace priors. For robust control, arbitrageurs make trading decisions under the criterion of *max-min expected utility*⁶.

My main finding is that model risk can motivate risk-neutral arbitrageurs to implement a machine-learning algorithm which mitigates their competition and ignores many mispricings. This result contains three points that are discussed in greater details below.

First, arbitrageurs' maximin robust strategy has a wide inaction zone: they start trading only when the observed order flow exceeds three standard deviations of noise trading. Yet this strategy is effective in catching the most profitable trades: arbitrageurs trade less than 2% of the time but can capture over 60% of the maximum profits they could earn in the absence of model risk. Under this strategy, arbitrageurs choose to ignore small mispricings. They focus on large events that involve little uncertainty about the trading direction. *Ex post*, an econometrician may find a lot of mispricings that persist in this economy and question arbitrageurs' rationality or capacity. In fact, arbitrageurs are rational and risk-neutral in my setting. They leave money on the table because of their aversion to *uncertainty*.

Second, this paper rationalizes a famous machine-learning method widely used in finance. The above-mentioned robust strategy is operationally equivalent to a simple algorithm called the *Least Absolute Shrinkage and Selection Operator* (LASSO)⁷. This is a powerful tool that can select a few key factors from a large set of regression coefficients. The standard statistical interpretation of LASSO involves a different mechanism, namely, the *Maximum a Posteriori* estimate. This learning rule lacks Bayesian rationality because it uses the posterior mode as point estimate, without summarizing all relevant information. In my setup, arbitrageurs are Bayesian-rational when they decide to use LASSO: they evaluate all possible states using Bayes' rule and dynamically maximize a well-defined utility with sequential rationality.

Third, the maximin robust strategy supports tacit collusion and impairs market efficiency. Arbitrageurs trade conservatively beyond the inaction zone. This enables them to accumulate market power, which is most prominent near the kinks of their robust strategy. Therefore, uncoordinated exercise of individual robust control facilitates tacit collusion among traders, without any communication device or explicit agreement. Remarkably, even as the number of arbitrageurs goes to infinity, their total profit does not vanish but converges to a finite level. This non-competitive payoff is due to the "cartel" effect which hinders price efficiency.

⁶The theory of max-min expected utility is a standard treatment for ambiguity-averse preferences. It is axiomatized by [Gilboa and Schmeidler \(1989\)](#), as a framework for robust decision making under uncertainty. Related discussions can be found in [Dow and Werlang \(1992\)](#) and [Hansen and Sargent \(2001\)](#) for example.

⁷LASSO is a machine-learning technique developed by [Tibshirani \(1996\)](#) to improve prediction accuracy and model interpretability. It is popular among algorithmic traders. This technique has recently been employed in many financial studies, such as [Huang and Shi \(2011\)](#), [Kozak, Nagel, and Santosh \(2017\)](#), [Chinco, Clark-Joseph, and Ye \(2017\)](#), and [Freyberger, Neuhierl, and Weber \(2017\)](#).

Finally, I extend the model to allow for strategic interaction between the informed trader and the group of arbitrageurs. The informed trader entices arbitrageurs to mimic past order flows; arbitrageurs' trend-following responses also tempt the informed trader to trick them: she may first trade a large quantity to trigger those arbitrageurs and then unwind her position against them. This strategy resembles several controversial schemes in reality. One example is *momentum ignition*, a trading algorithm that attempts to trigger many other algorithmic traders to run in the same direction so that the instigator can profit from trading against the momentum she ignited. Another scheme is *stop-loss hunting* which attempts to force some traders out of their positions by pushing the asset price to certain levels where they have set stop-loss orders. In my setup, this sort of strategies can impair pricing accuracy, exaggerate price volatility, and raise the average trading costs for common investors. Numerical results also generate empirically testable patterns regarding price overreactions and volatility spikes.

Contributions to the literature. This paper investigates strategic arbitrage trading in an uncertain fat-tail environment. This topic requires new methods and inspires fresh thinking. Results discussed in this paper can contribute in multiple ways to the vast literature of asset pricing, market microstructure, and behavioral finance.

First, this paper develops a new modeling framework for statistical arbitrage. The semi-strong-form market efficiency holds in the standard Kyle (1985) model where traders have common Gaussian beliefs about the economy. This simple assumption has been followed by most subsequent studies⁸. The present paper deviates from the literature by introducing fat-tail shocks to disrupt the Kyle equilibrium when market makers stick to Gaussian beliefs. Unexpected changes in the underlying distribution cause mispricings in the market. This gives room for arbitrageurs if they can foresee fat-tail shocks. Due to model risk, arbitrageurs are uncertain about the extent of mispricings. If they simply follow the maximin criterion, they may overemphasize the least favorable prior and become overly pessimistic in decision making. This paper implements a rational procedure that prevents such biases. Similar to the spirit of *rational expectations*, an internally consistent assumption is that arbitrageurs inside this model have the correct belief *on average* about the model structure, despite their uncertainty about some prior parameter. Recognizing this consistency, a rational arbitrageur only considers those strategies that converge to the optimal strategy (as averaged across all possible priors) and that preserve the convexity of their optimal strategy. Such constraints make their admissible strategies comparable to the ideal rational-expectations strategy⁹.

⁸The literature includes Back (1992), Holden and Subrahmanyam (1992), Foster and Viswanathan (1994), Foster and Viswanathan (1996), Vayanos (1999), Back, Cao, and Willard (2000), Vayanos (2001), Huddart, Hughes, and Levine (2001), and Collin-Dufresne and Fos (2016), among many others.

⁹The rational-expectations strategy is the one that traders would use if they knew the true Laplace prior.

Second, this paper is the first to study how market efficiency gets hindered by model risk when arbitrageurs have fat-tail beliefs. This angle distinguishes the present paper from the existing literature on *limits to arbitrage*¹⁰. Previous studies have suggested various important frictions, including short-selling costs, leverage constraints, and wealth effects, which limit arbitrageurs’ *ability* to eliminate mispricings. Excluding those frictions, the present paper identifies another mechanism that can strongly affect the *willingness* of arbitrageurs to trade. Specifically, model uncertainty of fat-tail priors make arbitrageurs hesitate to eradicate small mispricings, because of ambiguity about the trading direction.

Third, this work sheds light on interesting topics at the interface of behavioral finance and machine learning. This paper uses the max-min decision rule to rationalize the LASSO (“*soft-thresholding*”) strategy, which was taken by Gabaix (2014) as a behavioral assumption of the *anchoring-and-adjustment* mechanism. The LASSO algorithm has an inaction zone where agents choose to ignore whatever happened, similar to the *status quo bias*¹¹. The strategy of arbitrageurs also resembles the behavior of feedback traders discussed in behavioral finance¹². In the eyes of an observer who has a Gaussian prior, arbitrageurs are “irrational” because they show up randomly and all perform feedback trading based on historical prices. The observer’s view is incorrect, given his misspecified prior in this economy.

This study can also help us to interpret empirical results about high-frequency traders (HFTs)¹³. My primary model of statistical arbitrage can describe the situation where an informed institutional investor executed large orders over time without anticipating that HFTs detected her footprints to catch the momentum train; see Lewis (2014) for a historical account. As an extension, I consider strategic interaction between an informed trader and a group of arbitrageurs. This extended model can describe the situation where institutional investors anticipate those HFTs and optimize their execution algorithms with strategic considerations. My model is consistent with the empirical implications reported in van Kervel and Menkveld (2017) on HFTs around institutional trading: (1) “*HFTs appear to lean against the wind when an order starts executing but if it executes more than seven hours, they seem to reverse course and trade with wind.*” (2) “*Institutional orders appear mostly information-motivated, in particular the ones with long-lasting executions that HFTs eventually trade along with.*” (3) “*Investors are privately informed and optimally trade on their signal in full awareness of HFTs preying on the footprint they leave in the market.*”

¹⁰Gromb and Vayanos (2010) is an excellent survey on this subject. See also Shleifer and Vishny (1997), Xiong (2001), Gabaix, Krishnamurthy, and Vigneron (2007), Kondor (2009), among others.

¹¹See Kahneman, Knetsch, and Thaler (1991) and Samuelson and Zeckhauser (1988).

¹²For behavioral interpretations of feedback traders, see DeLong, Shleifer, Summers, and Waldmann (1990), Barberis, Greenwood, Jin, and Shleifer (2015), and Barberis, Greenwood, Jin, and Shleifer (2018).

¹³For recent research on high-frequency trading, see Hendershott, Jones, and Menkveld (2011), van Kervel and Menkveld (2017), Kirilenko, Kyle, Samadi, and Tuzun (2017), and Korajczyk and Murphy (2018).

The extended model also contributes to the body of literature on market manipulations¹⁴. In [Allen and Gale \(1992\)](#), a trade-based price manipulation is played by an *uninformed* trader who attempts to trick other traders into believing the existence of informed trading. In my model, the manipulative strategy is performed by an *informed* trader who trades in an unexpected way to distort the learning of other traders. The informed trader may hide her signal when it is strong and bluff when it is weak. In the linear equilibrium of [Foster and Viswanathan \(1994\)](#), the better informed trader may also hide her information in early periods and even trade against the direction of her superior signal. My analysis focuses on a nonlinear equilibrium where the informed trader hides her information to reduce competitive pressure from arbitrageurs. Several articles by [Chakraborty and Yilmaz](#)¹⁵ show that if market makers face uncertainty about the existence of informed trades, then the informed trader will bluff in every equilibrium by directly adding noise to other traders’ inference problem. The disruptive strategy in my model is different because (1) it occurs under a set of specific conditions, not state-by-state in every equilibrium; (2) it is a pure strategy that distorts the learning of other traders, not a mixed strategy that adds some noise¹⁶; (3) it produces bimodal distributions of prices, thereby magnifying both price volatility and trading costs.

Finally, the disruptive strategy in this paper shows that asset price “bubbles and crashes” can take place in a strategic environment where speculators have fat-tail beliefs. Under good enough liquidity conditions, a better-informed savvy trader may trade very aggressively to trigger those speculators whose subsequent momentum responses can give this savvy trader a reversal trading opportunity. This finding is related to the literature on market instability¹⁷. The mechanism here shares some similarity with the model of [Scheinkman and Xiong \(2003\)](#) where asset price bubbles reflect resale options due to traders’ overconfidence. In my setup, speculators’ over-aggressive trading implicitly grants the informed trader a “resale option” which could be exercised if condition permits. It is however worth remarking that traders in my (extended) model share a common fat-tail prior, without any overconfidence bias.

The rest of this paper is organized as follows. Section 2 focuses on the primary model where arbitrageurs exploit uncertain pricing errors in a robust manner. Section 3 studies the extended model where a savvy informed trader anticipates and exploits those arbitrageurs. Concluding remarks are made in Section 4. Major proofs are provided in Appendix A.

¹⁴See [Allen and Gale \(1992\)](#), [Kumar and Seppi \(1992\)](#), [Jarrow \(1992\)](#), [van Bommel \(2003\)](#), [Huberman and Stanzl \(2004\)](#), [Huddart et al. \(2001\)](#), [Khwaja and Mian \(2005\)](#), [Jiang, Mahoney, and Mei \(2005\)](#); [Brunnermeier \(2005\)](#), [Brunnermeier and Pedersen \(2005\)](#), [Kyle and Viswanathan \(2008\)](#), [Goldstein and Guembel \(2008\)](#), [Jarrow \(2015\)](#), and [Fox, Glosten, and Rauterberg \(2018\)](#).

¹⁵See [Chakraborty and Yilmaz \(2004a\)](#), [Chakraborty and Yilmaz \(2004b\)](#), [Chakraborty and Yilmaz \(2008\)](#).

¹⁶Mixed strategies are studied in modified Kyle models by [Huddart et al. \(2001\)](#) and [Yang and Zhu \(2017\)](#).

¹⁷See [Kyle and Xiong \(2001\)](#), [Abreu and Brunnermeier \(2003\)](#), [Hong and Stein \(2003\)](#), and [Scheinkman and Xiong \(2003\)](#), among others.

2 Model of Robust Arbitrageurs

In this section, an equilibrium model is developed to study how arbitrageurs' prior uncertainty about mispricing shocks affects arbitrage strategy and market efficiency. This model adds random fat-tail shocks to disturb the efficient market of a two-period Kyle (1985) model.

Table 1. The timeline and market participants in an economy of two auctions.

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Informed Trader	observe v	submit x_1	submit x_2	receive π_x
Noise Traders	...	submit u_1	submit u_2	...
Arbitrageurs	observe s	submit $z_{1,n}$	submit $z_{2,n}$	receive $\pi_{z,n}$
Market Makers	prior $\mathcal{N}(0, \sigma_v^2)$	set p_1	set p_2	observe v

Structure and Notation. Consider the market in Table 1 with two rounds of trading, indexed by $t = 1, 2$. The liquidation value of a risky asset, denoted \tilde{v} , is either Gaussian or Laplacian:

$$\tilde{v} = (1 - \tilde{s}) \cdot \tilde{v}_G + \tilde{s} \cdot \tilde{v}_L, \quad \text{where} \quad \tilde{v}_G \sim \mathcal{N}(0, \sigma_v^2), \quad \tilde{v}_L \sim \mathcal{L}(0, \xi_v), \quad \xi_v \equiv \frac{\sigma_v}{\sqrt{2}}. \quad (1)$$

Here, \tilde{s} takes the integer value 1 with probability α and takes the value 0 with probability $1 - \alpha$. The true Laplace scale parameter is set to be $\xi_v = \frac{\sigma_v}{\sqrt{2}}$ so that the variance of \tilde{v} is always σ_v^2 . The initial asset price is set as $p_0 = 0$ without loss of generality. The quantities traded by noise traders are Gaussian, denoted $\tilde{u}_1 \sim \mathcal{N}(0, \sigma_u^2)$ and $\tilde{u}_2 \sim \mathcal{N}(0, \gamma \sigma_u^2)$. The noise variances can be different, as tuned by the parameter $\gamma > 0$. All the random variables \tilde{v} , \tilde{s} , \tilde{u}_1 , and \tilde{u}_2 are mutually independent. The parameters $\{\sigma_v, \sigma_u, \gamma\}$ are common knowledge.

A risk-neutral informed trader privately observes \tilde{v} at $t = 0$, submits market orders, \tilde{x}_1 and \tilde{x}_2 , to buy or sell this asset before her private signal becomes public at $t = 3$. The strategy is denoted by a vector of real-valued functions, $\mathbf{X} = \langle X_1, X_2 \rangle$. Prices and volumes become public information right after the auctions take place. The information sets of informed trader before trading at $t = 1, 2$ are $\mathcal{I}_{1,x} = \{\tilde{v}\}$ and $\mathcal{I}_{2,x} = \{\tilde{v}, \tilde{p}_1\}$ where \tilde{p}_1 is the asset price at $t = 1$. It is justified to write $\tilde{x}_1 = X_1(\tilde{v})$ and $\tilde{x}_2 = X_2(\tilde{v}, \tilde{p}_1)$. The informed trader's total profit from both periods can be written as $\tilde{\pi}_x = \sum_{t=1}^2 (\tilde{v} - \tilde{p}_t) \tilde{x}_t$.

A number of risk-neutral arbitrageurs (indexed by $n = 1, \dots, N$) observe \tilde{s} , which encodes the distribution type of \tilde{v} . Each arbitrageur can place market orders, $\tilde{z}_{1,n}$ and $\tilde{z}_{2,n}$, to exploit potential market inefficiency. Their strategy profile is represented by a matrix of real-valued functions, $\mathbf{Z} = [\mathbf{Z}_1, \dots, \mathbf{Z}_N]$ where $\mathbf{Z}_n = \langle Z_{1,n}, Z_{2,n} \rangle$ is the n -th arbitrageur's strategy for $n = 1, \dots, N$. The information sets of arbitrageurs are $\mathcal{I}_{1,z} = \{\tilde{s}\}$ and $\mathcal{I}_{2,z} = \{\tilde{s}, \tilde{p}_1\}$ before their trading at $t = 1, 2$. The quantities traded by the n -th arbitrageur are $\tilde{z}_{1,n} = Z_{1,n}(\tilde{s})$ and $\tilde{z}_{2,n} = Z_{2,n}(\tilde{s}, \tilde{p}_1)$. The total profit for the n -th trader is denoted $\tilde{\pi}_{z,n} = \sum_{t=1}^2 (\tilde{v} - \tilde{p}_t) \tilde{z}_{t,n}$.

Uninformed competitive market makers clear the market by setting prices at which they strive to break even. Their pricing strategy is denoted by the vector of real-valued functions, $\mathbf{P} = \langle P_1, P_2 \rangle$. The total order flow $\tilde{y}_t \equiv \tilde{x}_t + \sum_{n=1}^N \tilde{z}_{t,n} + \tilde{u}_t$ is observed by market makers before they set the price \tilde{p}_t at period $t \in \{1, 2\}$. We can write $\tilde{p}_1 = P_1(\tilde{y}_1)$ and $\tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2)$.

Belief System. Several assumptions are needed to clarify traders' beliefs in this model:

Assumption 2.1. *The informed trader and market makers think that it was common belief among all traders that the asset liquidation value was normally distributed, $\tilde{v} \sim \mathcal{N}(0, \sigma_v^2)$.*

Assumption 2.2. *Arbitrageurs have the correct Gaussian prior when $\tilde{s} = 0$, but they face uncertainty about the variance of fat-tail shocks when $\tilde{s} = 1$. Their Laplace prior is modeled as $\mathcal{L}(0, \tilde{\xi})$ where $\tilde{\xi} \in \Omega$ is a positive random variable. Arbitrageurs are ambiguity-averse and maximize the minimum expected payoff over all possible priors. On average, arbitrageurs are correct about the information structure, despite their prior uncertainty.*

Assumption 2.3. *Arbitrageurs know that market makers and the informed trader obey Assumption 2.1. Moreover, Assumption 2.2 is held as common knowledge among arbitrageurs.*

Since fat-tail shocks occur with probability α in this market, the higher-order moments of \tilde{v} can differ from those of the Gaussian counterpart \tilde{v}_G . When $\alpha = 0$, the asset value \tilde{v} is exactly Gaussian and the model reduces to the standard two-period Kyle (1985) model. The Laplace probability density, $f_L(v) = \frac{1}{2\xi_v} \exp\left(-\frac{|v|}{\xi_v}\right)$, has fat tails as it decays to zero at an exponential rate. Thus, the likelihood of observing extreme events under the Laplace distribution is much higher than under the Gaussian distribution with identical variance.

Knowledge of \tilde{s} is valuable since it tells traders the distribution type of stock value. If market makers have fat-tail beliefs and observe $\tilde{s} = 1$, they should use a convex pricing rule (which is rarely seen in real data). The Gaussian prior in Assumption 2.1 permits linear pricing schedules compatible with empirical observations. Despite its simplicity, the linear pricing function can underestimate the fat-tail information in large order flows. This opens the door to arbitrageurs because market makers have mistakes with probability α .

Arbitrageurs are sophisticated traders who may use advanced technology to detect mispricings. Their privilege of observing \tilde{s} represents their superior ability to identify statistical arbitrage opportunities. Nonetheless, arbitrageurs often face uncertainty about their trading models. The failure of *Long-Term Capital Management* (LTCM) demonstrates the critical role of model risk and the disastrous impact when the worst-case scenario hit. This motivates Assumption 2.2 that arbitrageurs care about the worst-case expected profits for robustness. As proved by Gilboa and Schmeidler (1989), the *max-min expected utility* theory rationalizes ambiguity-averse preferences. However, decisions derived from maximin optimization tend to follow the least favorable prior regardless of its likelihood. This appears too pessimistic. A more realistic assumption is that arbitrageurs' admissible strategies converge, in a rational manner¹⁸, to the average of optimal strategies evaluated across all possible priors. Similar to the concept of *rational expectations*, I assume that arbitrageurs inside this model are correct on average about the model structure. Without systematic bias, the average of optimal strategies across all possible priors should converge to the *rational-expectations equilibrium* (REE) strategy which corresponds to the ideal case that they know the true prior ξ_v .

Assumptions 2.1, 2.2, and 2.3 capture salient features of real-life arbitrage. In a nearly efficient market, arbitrage opportunities should be rare and thus overlooked by most market participants. Such opportunities may be identified and exploited by a small number of traders (i.e., arbitrageurs who observe \tilde{s}). What may limit their trading is the model risk and their imperfect competition. Arbitrageurs are likely to have similar priors and preferences, given that they have similar forecasting technology and face similar pressures of robust control.

The belief system described in Assumption 2.1 can be denoted as $\mathcal{B} = \{\tilde{s} = 0\}$, which is shared by the informed trader and market makers. They think that it is common knowledge among all traders that $\tilde{v} \sim \mathcal{N}(0, \sigma_v^2)$. Arbitrageurs are aware of their Gaussian belief \mathcal{B} . By Assumptions 2.2 and 2.3, the belief system shared by arbitrageurs can be expressed as $\mathcal{A} = \{\tilde{s}, \tilde{\xi}\}$, where $\tilde{\xi}$ denotes the uncertain Laplace prior. Arbitrageurs' belief depends on the observed \tilde{s} which tells them the type of prior to use:

$$\tilde{v} \sim \mathcal{N}(0, \sigma_v^2) \text{ if } \mathcal{A} = \{\tilde{s} = 0, \tilde{\xi}\} \quad \text{and} \quad \tilde{v} \sim \mathcal{L}(0, \tilde{\xi}) \text{ if } \mathcal{A} = \{\tilde{s} = 1, \tilde{\xi}\}. \quad (2)$$

Obviously, \mathcal{A} and \mathcal{B} are consistent when $\tilde{s} = 0$ but they are at odds when $\tilde{s} = 1$. Market makers believe that any uninformed trader holds the same Gaussian prior as they do. In fact, arbitrageurs can infer that market makers use the wrong prior when $\tilde{s} = 1$.¹⁹

¹⁸To avoid overfitting, their admissible strategy should preserve the convexity of their optimal strategies.

¹⁹This is not "agreement to disagree" because traders have inconsistent belief structures here. Han and Kyle (2017) discussed the situation where traders have inconsistent beliefs about the mean. In my model, traders agree on the mean but hold inconsistent beliefs about higher moments of \tilde{v} .

2.1 Equilibrium Definition and Conjecture

The trading of arbitrageurs affects the realized profit of informed trader $\tilde{\pi}_x$. To emphasize its dependence on all traders' strategies, we write $\tilde{\pi}_x = \tilde{\pi}_x(\mathbf{X}, \mathbf{P}, \mathbf{Z})$. Similarly, each arbitrageur takes into account the strategies played by other traders. To stress such dependence, we write $\tilde{z}_{t,n} = \tilde{z}_{t,n}(\mathbf{X}, \mathbf{P}, \mathbf{Z})$ and $\tilde{\pi}_{z,n} = \tilde{\pi}_{z,n}(\mathbf{X}, \mathbf{P}, \mathbf{Z})$ for $n = 1, \dots, N$. By Assumption 2.2, each arbitrageur seeks to maximize the minimum expected profit over all possible priors:

$$\max_{\mathbf{Z}_n \in \mathcal{Z}^2} \min_{\xi \in \Omega} \mathbb{E}^{\mathcal{A}} \left[\tilde{\pi}_{z,n} \middle| \tilde{s}, \tilde{\xi} = \xi \right] = \max_{\mathbf{Z}_n \in \mathcal{Z}^2} \min_{\xi \in \Omega} \mathbb{E}^{\mathcal{A}} \left[\sum_{t=1}^2 (\tilde{v} - \tilde{p}_t) z_{t,n} \middle| \tilde{s}, \tilde{\xi} = \xi \right], \quad (3)$$

where $\mathbf{Z}_n = \langle z_{1,n}, z_{2,n} \rangle$. Both $z_{1,n}$ and $z_{2,n}$ are in the admissible set \mathcal{Z} which requires asymptotic convergence to the REE without losing the convexity/concavity of the REE strategy.

Definition of Equilibrium. A sequential trading equilibrium in this model is defined as a tuple of strategies $(\mathbf{X}, \mathbf{P}, \mathbf{Z})$ such that the following conditions hold:

1. For any alternative strategy $\mathbf{X}' = \langle X'_1, X'_2 \rangle$ differing from $\mathbf{X} = \langle X_1, X_2 \rangle$, the strategy \mathbf{X} yields an expected total profit no less than \mathbf{X}' , and also X_2 yields an expected profit in the second period no less than the single deviation X'_2 :

$$\mathbb{E}^{\mathcal{B}}[\tilde{\pi}_x(\mathbf{X}, \mathbf{P}, \mathbf{Z}) | \tilde{v}] \geq \mathbb{E}^{\mathcal{B}}[\tilde{\pi}_x(\mathbf{X}', \mathbf{P}, \mathbf{Z}) | \tilde{v}], \quad (4)$$

$$\mathbb{E}^{\mathcal{B}}[(\tilde{v} - \tilde{p}_2(\langle X_1, X_2 \rangle, \mathbf{P}, \mathbf{Z})) X_2 | \tilde{v}, \tilde{p}_1] \geq \mathbb{E}^{\mathcal{B}}[(\tilde{v} - \tilde{p}_2(\langle X_1, X'_2 \rangle, \mathbf{P}, \mathbf{Z})) X'_2 | \tilde{v}, \tilde{p}_1]. \quad (5)$$

2. For all $n = 1, \dots, N$ and any alternative strategy profile \mathbf{Z}' differing from \mathbf{Z} only in the n -th component $\mathbf{Z}'_n = \langle Z'_{1,n}, Z'_{2,n} \rangle$, the strategy profile \mathbf{Z} yields a utility level (i.e., the minimum expected profit over all possible priors) no less than \mathbf{Z}' , and also $Z_{2,n}$ yields a utility level in the second period no less than the single deviation $Z'_{2,n}$:

$$\min_{\xi \in \Omega} \mathbb{E}^{\mathcal{A}}[\tilde{\pi}_{z,n}(\mathbf{X}, \mathbf{P}, \mathbf{Z}) | \tilde{s}, \tilde{\xi} = \xi] \geq \min_{\xi \in \Omega} \mathbb{E}^{\mathcal{A}}[\tilde{\pi}_{z,n}(\mathbf{X}, \mathbf{P}, \mathbf{Z}') | \tilde{s}, \tilde{\xi} = \xi]; \quad (6)$$

$$\min_{\xi \in \Omega} \mathbb{E}^{\mathcal{A}}[(\tilde{v} - \tilde{p}_2(\cdot, Z_{2,n})) Z_{2,n} | \tilde{s}, \tilde{p}_1, \tilde{\xi} = \xi] \geq \min_{\xi \in \Omega} \mathbb{E}^{\mathcal{A}}[(\tilde{v} - \tilde{p}_2(\cdot, Z'_{2,n})) Z'_{2,n} | \tilde{s}, \tilde{p}_1, \tilde{\xi} = \xi] \quad (7)$$

where the strategy profile on the right hand side of Eq. (7) only differs from $(\mathbf{X}, \mathbf{P}, \mathbf{Z})$ at $Z_{2,n}$. Any strategy considered by arbitrageurs has to be in the admissible set \mathcal{Z} .

3. The prices, $\mathbf{P} = \langle P_1, P_2 \rangle$, are set by the market makers' posterior expectation of \tilde{v} :

$$\tilde{p}_1 = P_1(\tilde{y}_1) = \mathbb{E}^{\mathcal{B}}[\tilde{v} | \tilde{y}_1], \quad \text{and} \quad \tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2) = \mathbb{E}^{\mathcal{B}}[\tilde{v} | \tilde{y}_1, \tilde{y}_2]. \quad (8)$$

Equilibrium Conjecture. The full equilibrium $(\mathbf{X}, \mathbf{P}, \mathbf{Z})$ can be characterized separately. The informed trader and market makers believe that they were living in a two-period Kyle model (Assumption 2.1). They think that arbitrageurs held the same Gaussian belief and would not trade in a conjectured equilibrium with (semi-strong-form) market efficiency. This inspires them to conjecture a subgame perfect linear equilibrium (\mathbf{X}, \mathbf{P}) .

Proposition 2.1. *Under Assumptions 2.1, there exists a unique subgame perfect linear equilibrium (\mathbf{X}, \mathbf{P}) identical to the linear equilibrium of a two-period Kyle (1985) model with normally distributed random variables. Market makers set the linear pricing rule:*

$$\tilde{p}_1 = P_1(\tilde{y}_1) = \lambda_1 \tilde{y}_1, \quad \tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2) = \tilde{p}_1 + \lambda_2 \tilde{y}_2, \quad \lambda_1 = \frac{\sqrt{2\delta(2\delta-1)} \sigma_v}{4\delta-1}, \quad \lambda_2 = \delta \lambda_1. \quad (9)$$

The equilibrium ratio $\delta = \frac{\lambda_2}{\lambda_1}$ is determined by the largest root to the cubic equation:

$$8\gamma\delta^3 - 4\gamma\delta^2 - 4\delta + 1 = 0. \quad (10)$$

The informed trader follows the linear trading strategy:

$$\tilde{x}_1 = X_1(\tilde{v}) = \frac{\tilde{v}}{\rho\lambda_1} = \frac{2\delta-1}{4\delta-1} \cdot \frac{\tilde{v}}{\lambda_1}, \quad \tilde{x}_2 = X_2(\tilde{v}, \tilde{y}_1) = \frac{\tilde{v} - \lambda_1 \tilde{y}_1}{2\delta\lambda_1}, \quad (11)$$

where $\rho \equiv \frac{4\delta-1}{2\delta-1}$ is a liquidity-dependent parameter that reflects the trading intensity at $t = 1$. Informed trader and market makers believe that no arbitrageurs would trade under (\mathbf{X}, \mathbf{P}) .

Proof. This is an extension of Proposition 1 in Huddart et al. (2001). See Appendix A.1. \square

To break even under different liquidity conditions, market makers can adjust the slopes of linear pricing schedules. For example, when noise trading volatility is constant (i.e., $\gamma = 1$), they can solve from Eq. (10) that $\delta \approx 0.901$; when $\gamma = \frac{3}{4}$, they can find that $\delta = 1$ and $\lambda_1 = \lambda_2 = \frac{\sqrt{2}}{3} \frac{\sigma_v}{\sigma_u}$; when liquidity evaporates ($\gamma \rightarrow 0$), the solution explodes: $\delta \rightarrow \infty$ so that $\lambda_1 = \frac{\sigma_v}{2\sigma_u}$ and $\lambda_2 \rightarrow \infty$. It is convenient to introduce a dimensionless parameter to denote the liquidity condition. Market depth is usually measured by the inverse of price impact parameter. To quantify the change of market depth in the second period, I define

$$\mu \equiv \frac{\lambda_1^{-1} - \lambda_2^{-1}}{\lambda_1^{-1}} = 1 - \frac{1}{\delta}. \quad (12)$$

In general, $\mu \in [-1, 1]$. For example, $\mu = 0.5$ indicates a 50% drop of market depth, while $\mu = 0$ reflects constant depth. Market depth becomes higher (i.e., $\mu < 0$) if $\gamma > \frac{3}{4}$.

If market makers know that \tilde{v} is drawn from the mixture distribution, the linear pricing rule in Eq. (9) can still help them to break even, regardless of the mixture parameter α . Linear pricing preserves the symmetry of probability distributions so that market makers' unconditional expected profits are zero : $E[(\tilde{p}_2 - \tilde{v})\tilde{y}_2] = 0$ and $E[(\tilde{p}_1 - \tilde{v})\tilde{y}_1 + (\tilde{p}_2 - \tilde{v})\tilde{y}_2] = 0$. This shows the robustness of linear pricing strategy and may explain its popularity.

By Proposition 2.1, the informed trader and market makers believe that no arbitrageurs would trade in this market. Thus, any strategy profile \mathbf{Z} does not affect the linear equilibrium strategies \mathbf{X} and \mathbf{P} . Arbitrageurs can take Proposition 2.1 as given when solving their own dynamic optimization problems Eq. (6) and Eq. (7). Arbitrageurs know that the informed trader and market makers do not anticipate their trading. Arbitrageurs take into account the price impacts of all traders in the market. When $s = 0$, the belief structure of all traders is consistent and correct. In this case, arbitrageurs have no advantage over market makers.

Corollary 2.1. *When $s = 0$, arbitrageurs do not trade because the market is indeed efficient.*

Arbitrageurs are better “informed” than market makers in the presence of fat-tail shocks. Will they trade immediately? Let us conjecture now and verify later that arbitrageurs would not trade in the first period. This is intuitive given the symmetry of their priors and the linearity of pricing rule. It simplifies the procedure to solve this equilibrium. First, Eq. (7) can be used to derive the optimal strategy profile $\langle Z_{2,1}, \dots, Z_{2,N} \rangle$ in the next period under the conjecture that $Z_{1,n} = 0$ for all $n = 1, \dots, N$. Second, Eq. (6) can be used to verify that it is not a profitable deviation for any arbitrageur to trade in the first period. If no one would deviate, $\mathbf{Z} = [\langle 0, Z_{2,1} \rangle, \dots, \langle 0, Z_{2,N} \rangle]$ will indeed be the equilibrium strategy for arbitrageurs.

2.2 Optimal Strategy without Model Risk

The linearity of informed trader's strategy $X_1(v) = \frac{v}{\rho\lambda_1}$ simplifies arbitrageurs' inference. Intuitively, the quantities traded by them in the presence of fat-tail shocks are proportional to their conditional expectation of the stock value mispriced by the market. Of course, the posterior estimate of \tilde{v} depends on their fat-tail priors. It is helpful to study the ideal case that model risk vanishes. If there is no ambiguity in their prior, arbitrageurs become *subjective expected utility* optimizers, under their Laplace prior $\mathcal{L}(0, \xi)$ when $s = 1$.

Proposition 2.2. *In the absence of model risk, arbitrageurs maximize their expected profits. Over the liquidity regime $\mu > \mu_\epsilon \approx -0.2319$ where μ_ϵ is the largest root to the cubic equation $\mu^3 + 21\mu^2 + 35\mu + 7 = 0$, arbitrageurs do not trade at $t = 1$ and their optimal strategy at*

$t = 2$ is proportional to their posterior expectation of $\tilde{\theta} = \tilde{v} - p_1$ under the prior $\mathcal{L}(0, \xi)$:

$$Z_{2,n}^o(s, y_1; \xi) = s \frac{1 - \mu}{N + 1} \cdot \frac{\hat{v}(y_1; \xi) - \lambda_1 y_1}{2\lambda_1} = s \frac{1 - \mu}{N + 1} \cdot \frac{\hat{\theta}(y_1; \xi)}{2\lambda_1}, \quad n = 1, \dots, N. \quad (13)$$

The estimator $\hat{v}(y_1; \xi)$ is the posterior mean of \tilde{v} under the prior that \tilde{v} is drawn from $\mathcal{L}(0, \xi)$:

$$\hat{v} = E^A[\tilde{v} | y_1 = y' \sigma_u, \xi] = \frac{\kappa \xi (y' - \kappa) \operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right)}{\operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right) + e^{2\kappa y'} \operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right)} + \frac{\kappa \xi (y' + \kappa) \operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right)}{\operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right) + e^{-2\kappa y'} \operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right)}. \quad (14)$$

The rescaled estimator \hat{v}/ξ is an increasing function of the rescaled quantity $y' = y_1/\sigma_u$, with one dimensionless shape parameter, $\kappa \equiv \frac{\rho \lambda_1 \sigma_u}{\xi}$. The rational-expectations equilibrium (REE) corresponds to the case that their prior is correct, i.e., $\xi = \xi_v$. Under REE, $\kappa = \frac{2}{\sqrt{1+\mu}}$.

Proof. See Appendix A.2. □

Arbitrageurs only trade when fat-tail shocks occur. In the eyes of some econometrician who holds the Gaussian belief and trusts in market efficiency, those arbitrageurs seem to be “irrational” because they show up randomly and behave like feedback traders. This may raise various behavioral arguments, without recognizing the misspecification of priors.

Arbitrageurs’ prior is symmetric (non-directional) at the beginning. They postpone arbitrage trading until they could tell the trading direction from past price movements, or equivalently, until their posterior beliefs become skewed. Proposition 2.2 confirms this no-trade conjecture in the first period. It also explains why this paper starts from a two-period setup. Even though arbitrageurs are better informed (with the knowledge of \tilde{s}) than market makers, their prior expectation of the stock value is identical to market makers’. Arbitrageurs have to watch the market first to see in which direction market makers incur pricing errors. This “wait-and-see” strategy suggests that arbitrage trading can be delayed for learning purposes so that mispricings may sustain for a longer period of time. The mechanism here is different from the delayed arbitrage discussed in [Abreu and Brunnermeier \(2002\)](#) where arbitrageurs face uncertainty about when their peers will exploit a common arbitrage opportunity.

The optimal strategy is symmetric with the past order flow: $Z_{2,n}^o(s, -y_1) = -Z_{2,n}^o(s, y_1)$. The rescaled strategy, $Z_{2,n}^o/\sigma_u$, is a function of the rescaled order flow $y' = y_1/\sigma_u$ in the fat-tail case. The optimal strategy becomes almost linear at large order flows. Its asymptotic slope is equal to the slope of linear strategy for traders who have a uniform prior ($\xi \rightarrow \infty$). Examination of the first and second derivatives leads to the following statement.

Corollary 2.2. *When $s = 1$, the optimal strategy $Z_{2,n}^o(s, y_1)$ is convex in the positive domain of y_1 and concave otherwise. It is asymptotically linear with a limit slope of $\frac{1-\mu}{(1+\mu)(N+1)}$.*

2.3 Robust Strategy under Model Risk

As indicated by Eq. (14), the estimator \hat{v} depends on the dispersion of Laplace prior, ξ . How would arbitrageurs trade when they have uncertain priors? Model risk is a critical issue in statistical arbitrage, because using a wrong prior could yield a business disaster like the failure of LTCM. In the real world, traders often face the pressure to test the performance of their strategies in the worst-case scenario. This pressure can drive them to adopt alternative strategies that sacrifice some optimality for robustness.

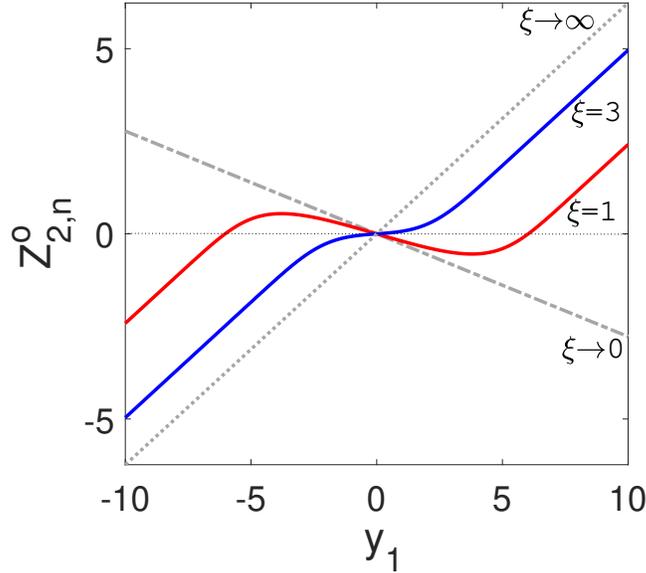


Figure 1. The optimal strategy $Z_{2,n}^o(s = 1, y_1; \xi)$ in Eq. (13) under different values of ξ .

Fig. 1 shows the optimal strategy under different values of the Laplace parameter ξ . An arbitrageur with the prior $\xi \rightarrow 0$ believes that the stock value is unchanged (i.e., $\tilde{v} = 0$). This trader will attribute all the order flow y_1 to noise trading and trade against any price change. In contrast, an arbitrageur with the extreme prior $\xi \rightarrow \infty$ believes that the past order flow is dominated by informed trading and thus will chase the price trend straightly. For small ξ , arbitrageurs will engage in contrarian trading on small order flows which are dominated by noise trading under their belief. For large ξ , arbitrageurs always use a momentum strategy.

Suppose that arbitrageurs' uncertain prior $\tilde{\xi}$ is in the interval $[\xi_L, \xi_H]$, where both the highest and lowest priors, ξ_H and ξ_L , have non-zero chances. If the divergence between ξ_H and ξ_L is large enough, arbitrageurs can face ambiguity about the trading direction conditional on small order flows²⁰: they may want to buy the asset under a high prior (for example,

²⁰If the extreme priors satisfy $y_1 Z_{2,n}^o(s, y_1; \xi_H) > 0$ for any $y_1 \neq 0$ and $y_1 Z_{2,n}^o(s, y_1; \xi_L) \leq 0$ for a nonzero measure of y_1 , then different fat-tail priors can give opposite trading directions at small order flows.

$\xi = 3$ in Fig. 1) but sell it under a low prior (for example, $\xi = 1$ in Fig. 1). If they use the wrong prior, they may trade in the wrong direction and undergo adverse fat-tail shocks.

By Assumption 2.2, arbitrageurs rank strategies based on the maximin decision criterion, i.e., each arbitrageur maximizes the minimum expected profit over a set of multiple priors. Pure maximin optimization can give very pessimistic decisions which stick to the least favorable prior even if it has a tiny chance to occur. To avoid over-pessimistic responses, I assume that arbitrageurs' admissible strategies converge to the averaged optimal strategy (across all priors) in a rational manner that preserves its convexity and/or concavity. Let's also enforce internal consistency: arbitrageurs inside this model "know" its structure in a statistical sense. On average, they are correct about the economy without systematic bias.

First, it is reasonable and important to invoke the convergence condition. If arbitrageurs observe an extremely large order flow y_1 , they will be pretty sure that y_1 was dominated by informed trading in the fat-tail scenario. This resolves their ambiguity about trading directions and boosts their confidence to follow the averaged optimal strategy, $E^A[Z_{2,n}^o(\tilde{s}, y_1; \tilde{\xi}) | \tilde{s} = 1]$. Let Z^∞ denote the asymptotes of this averaged strategy. Simple derivation yields

$$Z^\infty(y_1, K_\xi) = \frac{1 - \mu}{1 + \mu} \cdot \frac{y_1 - \text{sign}(y_1)K_\xi}{N + 1}, \quad \text{where } K_\xi = \frac{\lambda_1 \rho^2 \sigma_u^2}{\rho - 1} E^A[\tilde{\xi}^{-1}]. \quad (15)$$

To ensure internal consistency, Eq. (15) should coincide with the asymptotes of the *rational expectations equilibrium* (REE) strategy given the true prior ξ_v . This requires $E^A[\tilde{\xi}^{-1}] = \xi_v^{-1}$ under which the asymptotes becomes $Z^\infty(y_1, K^*)$ where

$$K^* = \frac{\lambda_1 \rho^2 \sigma_u^2}{(\rho - 1)} \xi_v^{-1} = \frac{3 + \mu}{\sqrt{1 + \mu}} \sigma_u = \frac{\sqrt{2} \sigma_v}{\lambda_1}. \quad (16)$$

The condition $E^A[\tilde{\xi}^{-1}] = \xi_v^{-1}$ means that arbitrageurs' average belief is correct regarding the *precision* of Laplace prior. Similar to the concept of *rational expectations*, arbitrageurs inside this model make unbiased predictions on average, despite their uncertainty about the model structure. Any candidate strategy should converge to $Z^\infty(y_1, K^*)$. This condition ensures that the strategy space of arbitrageurs is anchored to their REE strategy (benchmark).

Second, the admissible strategies should rationally preserve the convexity and/or concavity of the optimal strategy. By Corollary 2.2, any optimal strategy (without model risk) is convex in the positive domain and concave otherwise (Fig. 1). Thus, any candidate strategy must be convex in the regime of $y_1 > 0$ and concave in the regime of $y_1 < 0$. Without this convexity-preserving condition, traders would consider strategies with arbitrarily complex curvatures. This may cause over-fitting problems and make model interpretation difficult.

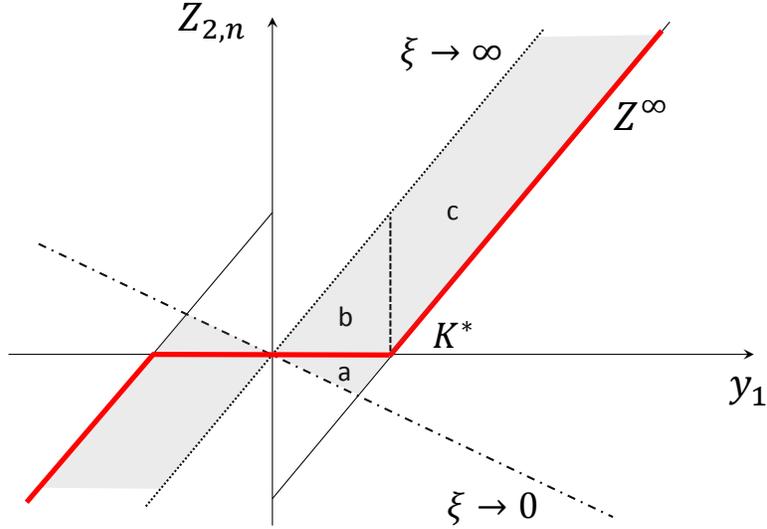


Figure 2. The robust strategy $Z_{2,n}$ in the presence of model risk.

Any strategy that converges to the REE strategy without losing its convex property must lie in the shaded areas of Fig. 2. Any strategy running outside this area violates either the convergence condition or the convexity-preserving rule. We can focus on the positive domain and divide the shaded area into three regions. For any $y_1 \in [0, K^*]$, arbitrageurs will not sell against y_1 , because they may lose money if the highest prior ξ_H is true. This rules out any decision point inside the triangle “a”. Similarly, arbitrageurs will not buy the stock since they may also lose money if the lowest prior ξ_L is true. This rules out any decision point inside the triangle “b”. So the max-min choice criteria indicate a no-trade zone over $y_1 \in [0, K^*]$. Next, for any $y_1 > K^*$, ambiguity-averse traders should not trade a quantity more than the one prescribed by the REE asymptotes $Z^\infty(y_1, K^*)$; otherwise they may lose in the worst-case scenario. This argument rules out any decision point inside the region “c”. By symmetry, the robust strategy turns out to be a piecewise linear function of y_1 , with the trading threshold K^* . This simple strategy is labeled by the red line in Fig. 2.

Proposition 2.3. *If arbitrageurs face sufficient model uncertainty about the fat-tail priors and if they follow the max-min choice criteria to rank the admissible strategies defined before, then their robust strategy at $t = 2$ is a piece-wise linear function of the order flow at $t = 1$:*

$$Z_{2,n}(s, y_1; K^*) = sZ^\infty(y_1, K^*)\mathbf{1}_{|y_1|>K^*} = s\frac{1-\mu}{1+\mu} \cdot \frac{y_1 - \text{sign}(y_1)K^*}{N+1} \cdot \mathbf{1}_{|y_1|>K^*}, \quad (17)$$

which is along the REE asymptotes with the trading threshold K^* given by Eq. (16).

Proof. See Appendix A.3. □

The endogenous decision boundary K^* is independent of the number of arbitrageurs (N) or the variance of asset value (σ_v^2). For constant noise trading volatility ($\gamma = 1$), one can find $K^* \approx 3.063\sigma_u$ which is roughly three standard deviations of noise order flows. This indicates a very large inaction zone for the robust strategy. To see how inactive it is, let us examine the unconditional variance of the first-period total order flow, $\sigma_y^2 = \frac{\sigma_v^2}{(\rho\lambda_1)^2} + \sigma_u^2 = \frac{3+\mu}{2}\sigma_u^2$, which implies $K^* \approx 2.5483\sigma_y$. When the asset value \tilde{v} is Laplacian, the probability that arbitrageurs get triggered to trade is very small, $P(|y_1| > K^*) \approx 1.33\%$. One might think that such a strategy is too inert to be profitable. This is not true. Numerically, the robust strategy can capture about 60% of the maximum profit recouped by the ideal REE strategy. This performance is surprisingly good given the idleness of the robust strategy. Fat-tail shocks create a disproportionate distribution of mispricings. The robust strategy is effective in picking up most profitable opportunities which correspond to those large fat-tail events.

So far, I have discussed various belief-related reasons for arbitrageurs' inaction. Their no-trade conditions are summarized as follows:

Corollary 2.3. *Arbitrageurs do not trade if any of the following conditions holds:*

- (1) *the market is efficient in the semi-strong form under their belief;*
- (2) *their prior expectation of \tilde{v} is identical to market makers' expectation;*
- (3) *the past price change cannot drive them out of their inaction (ambiguity) zone.*

Proof. Condition (1) holds at $\tilde{s} = 0$, Condition (2) holds for their decision making at $t = 1$, and Condition (3) is implied by Proposition 2.3. \square

Given their idleness, it may well be the case that arbitrageurs are overlooked by the rest of the market. This is self-consistent with the implication of Assumptions 2.1, 2.2, and 2.3.

More importantly, given their no-trade strategy in the first period and inaction region in the second period, a lot of pricing errors can persist in this market. *Ex post*, an econometrician can run regressions on historical data to discover many mispricings in this economy. The econometrician may question the rationality or capability of arbitrageurs as they apparently leave money on the table. *Ex ante*, arbitrageurs assess all possible states using Bayes' rule. They are risk-neutral but ambiguity averse. For maximin robustness, they rationally ignore small profit opportunities which involve ambiguity about the trading direction. Neither financial constraints nor trading frictions exist here. There is no limit to arbitrageurs' trading ability. It is model risk that reduces their willingness to eliminate mispricings. This intrinsic friction is especially important in the fat-tail world where it leads to a large no-trade zone.

2.4 Equivalent Learning Rule and Alternative Interpretations

The optimal strategy without model risk uses the posterior *mean* estimate in Bayesian learning (Proposition 2.2). What is the learning mechanism behind the robust strategy? Arbitrageurs are Bayesian rational when they solve their maximin objectives, Eq. (6) and Eq. (7). It is noteworthy that the derived (robust) strategy is observationally equivalent to the *Least Absolute Shrinkage and Selection Operator* (LASSO), a famous machine-learning technique developed by Tibshirani (1996). The LASSO estimate can be interpreted as the posterior *mode* under independent Laplace prior. In statistics, the posterior *mode* is formally known as the *Maximum a Posteriori* (MAP) estimate. This learning rule itself lacks Bayesian rationality because it does not use all relevant information in forming expectations of unknown variables²¹. Nonetheless, the MAP estimate can “produce” the robust strategy.

Proposition 2.4. *If arbitrageurs know the true Laplace prior ξ_v but directly use the MAP learning rule to estimate the mispricing signal $\tilde{\theta} = \tilde{v} - p_1$, then their strategy in the second period will be operationally equivalent to the robust strategy in Proposition 2.3:*

$$Z_{2,n}(s = 1, y_1; K^*) = \frac{\hat{\theta}_{map}}{2(N + 1)\lambda_2} = \frac{(\hat{v}_{map} - \lambda_1 y_1)\mathbf{1}_{|y_1| > K^*}}{2(N + 1)\lambda_2}. \quad (18)$$

Here, $\hat{\theta}_{map}$ is the MAP estimate of $\tilde{\theta}$. It contains \hat{v}_{map} which is the MAP estimate of \tilde{v} under the prior $\mathcal{L}(0, \xi_v)$. This is a soft-thresholding function with a threshold $\kappa\sigma_u = \frac{\rho\lambda_1\sigma_u^2}{\xi_v} = \frac{2\sigma_u}{\sqrt{1+\mu}}$:

$$\hat{v}_{map}(y_1; \xi_v) = \rho\lambda_1[y_1 - \text{sign}(y_1)\kappa\sigma_u]\mathbf{1}_{|y_1| > \kappa\sigma_u}. \quad (19)$$

Proof. See Appendix A.4. □

Fig. 3 compares the learning rules and their associated strategies. Both the posterior mean estimate \hat{v} and the REE strategy $Z_{2,n}^o(s = 1, y_1; \xi_v)$ are smooth and nonlinear. In contrast, the posterior mode estimate \hat{v}_{map} is zero for $y_1 \in [-\kappa\sigma_u, \kappa\sigma_u]$ and linear beyond that zone. The robust strategy $Z_{2,n}$ has a similar pattern as it performs linear momentum trading beyond the inaction zone $[-K^*, K^*]$. Traders who follow this strategy only respond to large events and deliberately ignore small ones. This rational response is similar to various behavioral patterns, including limited attention, status quo bias, anchoring and adjustment, among others²². Again, it is worth stressing that arbitrageurs are Bayesian-rational here: they evaluate all possible states using Bayes rule and maximize their well-defined utility with

²¹The MAP estimate of a variable equals the mode of the posterior distribution. As a point estimate, it does not summarize all relevant information in the posterior distribution.

²²Barberis and Thaler (2003) provide an excellent survey on those topics in behavioral finance.

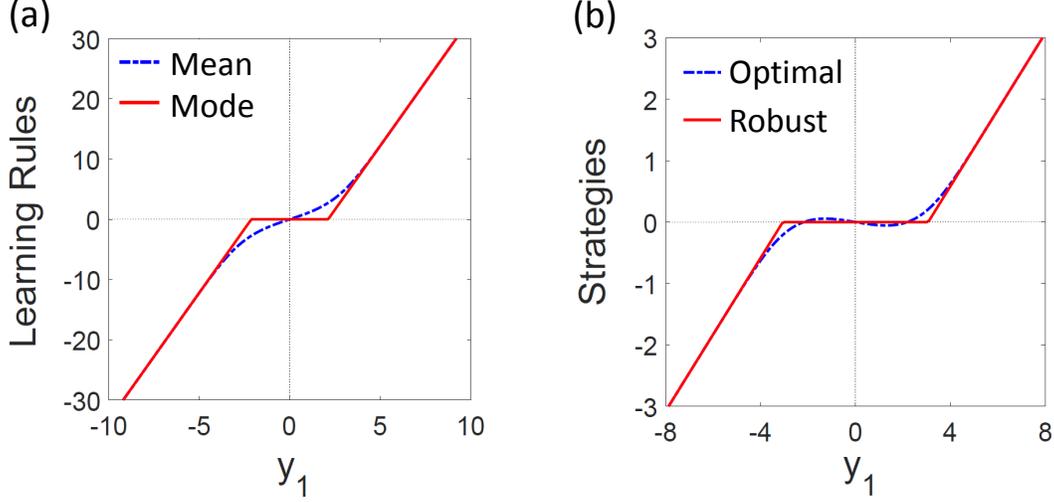


Figure 3. (a) The posterior mean versus the posterior mode of \tilde{v} under the Laplace prior $\mathcal{L}(0, \xi_v)$. (b) the optimal (REE) strategy versus the robust strategy at $t = 2$ when $s = 1$.

sequential rationality. One can apply Propositions 2.3 and 2.4 to rationalize the behavioral assumption of Gabaix (2014). In his model, the *soft-thresholding* function like Eq. (19) is used to describe the anchoring bias. Such behavior also permits a rational interpretation.

In a multi-asset economy subject to uncertain fat-tail shocks, Proposition 2.4 implies that arbitrageurs can directly incorporate the LASSO algorithm into their trading system:

Corollary 2.4. *Suppose that arbitrageurs identify $M \geq 1$ assets with independent and identically distributed liquidation values, $\tilde{v}_i \sim \mathcal{L}(0, \xi_v)$ for $i = 1, \dots, M$, and each of these assets is traded by a single informed trader in the two-period Kyle model with constant noise trading. For robust learning, arbitrageurs solve the LASSO objective in the Lagrangian form:*

$$\min_{\{v_1, \dots, v_M\}} \sum_{i=1}^M \left| p_{1,i} - \frac{v_i}{\rho} \right|^2 + \frac{2(\lambda_1 \sigma_u)^2}{\xi_v} |v_i|, \quad (20)$$

where $p_{1,i} = \lambda_1 y_{1,i}$ is the price change of the i -th asset and ρ^{-1} is the percentage of private signal that has been incorporated into the asset price at $t = 1$. This leads to a simple strategy

$$Z_{2,n}(p_{1,i}, \xi_v) = \frac{\rho - 1}{N + 1} \cdot \frac{p_{1,i} \pm 2\xi_v}{2\lambda_2} \cdot \mathbf{1}_{|p_{1,i}| \geq 2\xi_v}, \quad \text{for } i = 1, \dots, M, \quad (21)$$

which is automatically triggered to trade the i -th asset if its price change $p_{1,i}$ exceeds $\pm 2\xi_v$.

Proof. See Appendix A.4. □

The objective of maximizing the posterior (under MAP) is equivalent to the minimization problem Eq. (20). It involves an l^1 penalty term that comes from the Laplace prior $\mathcal{L}(0, \xi_v)$. LASSO shrinks certain estimation coefficients to zero and effectively selects a simpler model that exclude those coefficients. This is a popular tool among quantitative traders because it picks up a small number of key features (factors) from a large set of candidate features. For traders who use LASSO, their trading models shall involve fat-tail (typically Laplace) priors. If traders use the Gaussian prior instead, they will incur an l^2 penalty in their objective. The resulted algorithm is *ridge regression* which uniformly shrinks the size of all coefficients but does not send any coefficients to zero. Even with parameter uncertainty about the Gaussian prior, traders will not get an inaction zone. This is because signal inference is linear when the posterior is Gaussian. For symmetric unimodal distributions, the mean coincides with the mode; the two learning rules will give identical predictions. Since different Gaussian priors only change the slopes of linear responses, the maximin robust strategy in a pure Gaussian-mixture model will be linear; see Appendix A.4 for more details.

Corollary 2.4 can help explain the momentum strategy and anomaly in asset pricing²³. Short-term momentum traders can be viewed as statistical arbitrageurs who have uncertain fat-tail priors about mispriced stocks. Their robust trading is exactly the momentum strategy of buying winners and selling losers. Those traders usually focus on top market gainers and losers, instead of the entire universe of equities. Corollary 2.4 can also be used to interpret rule-based algorithmic trading which gets triggered at some predefined price levels. At first glance, such trading behavior seems to be mechanical and at odds with Bayesian rationality. It is possible that algorithmic traders are Bayesian-rational. They may use machine-learning techniques (such as LASSO) to manage unknown risks or improve prediction accuracy.

The robust LASSO strategy can also be used by market makers for error self-correction. Market makers can split their pricing logic into two programs. The first one is the linear pricing strategy which allows them to almost break even, despite their occasional mistakes. The second program uses the fat-tail prior to correct the errors of linear pricing strategy, just like the actions of arbitrageurs. This leads to the LASSO algorithm. Integrating both programs, market makers can keep using the linear pricing rule until their inventory exceeds the endogenous thresholds. At that point, they will switch to momentum trading and reduce excessive inventories. The no-trade zone in the second program is the ambiguity zone where they hesitate to correct uncertain pricing errors; this no-trade zone is also their comfortable zone to do market making. This new interpretation differs from conventional arguments that market makers' inventory limits are due to their high risk aversion or large inventory costs.

²³See Jegadeesh and Titman (1993), Chan, Jegadeesh, and Lakonishok (1996), Carhart (1997), Hong and Stein (1999), Daniel, Hirshleifer, and Subrahmanyam (1998), Lee and Swaminathan (2000), among others.

2.5 Cartel Effect and Market Inefficiency

Arbitrageurs trade conservatively beyond the endogenous inaction zone. Their conservative trading facilitates their tacit collusion which mitigates their competition and impedes market efficiency. This has interesting implications for limits to arbitrage.

Proposition 2.5. *As $N \rightarrow \infty$, the total profit of arbitrageurs vanishes if they use the REE strategy. However, their total profit has a positive limit if they follow the robust strategy.*

Proof. If arbitrageurs all follow the optimal REE strategy $Z_{2,n}^o(s, y_1; \xi_v)$, they will compete away their total arbitrage profit when N goes to infinity:

$$\begin{aligned}
\lim_{N \rightarrow \infty} \mathbb{E}^A \left[\sum_{n=1}^N (\tilde{v} - \tilde{p}_2) Z_{2,n}^o \right] &= \lim_{N \rightarrow \infty} \mathbb{E}^A \left[N (\tilde{v} - \lambda_1 \tilde{y}_1 - \lambda_2 X_2(\tilde{v}, \tilde{y}_1) - N \lambda_2 Z_{2,n}^o) Z_{2,n}^o \right] \\
&= \lim_{N \rightarrow \infty} \mathbb{E}^A \left[\frac{(N+1)(\tilde{v} - \lambda_1 \tilde{y}_1) - N(\hat{v} - \lambda_1 \tilde{y}_1)}{2(N+1)} \cdot \frac{N(\hat{v} - \lambda_1 \tilde{y}_1)}{2(N+1)\lambda_2} \right] \\
&= \lim_{N \rightarrow \infty} \frac{N}{4(N+1)^2 \lambda_2} \mathbb{E}^A [(\hat{v}(\tilde{y}_1) - \lambda_1 \tilde{y}_1)^2] = 0, \tag{22}
\end{aligned}$$

where in the above derivation we have used Eq. (11) and $\mathbb{E}^A[\tilde{v}] = \mathbb{E}^A[\mathbb{E}^A[\tilde{v}|\tilde{y}_1]] = \mathbb{E}^A[\hat{v}(\tilde{y}_1)]$.

In contrast, if arbitrageurs follow the robust strategy $Z_{2,n}(s, y_1; K^*)$, their total arbitrage profit will converge to a positive value, indicating a cartel effect:

$$\begin{aligned}
\lim_{N \rightarrow \infty} \mathbb{E}^A \left[\sum_{n=1}^N (\tilde{v} - \tilde{p}_2) Z_{2,n} \right] &= \lim_{N \rightarrow \infty} \mathbb{E}^A \left[\frac{(N+1)(\tilde{v} - \lambda_1 \tilde{y}_1) - N\hat{\theta}_{map}}{2(N+1)} \cdot \frac{N\hat{\theta}_{map}}{2(N+1)\lambda_2} \right] \\
&= \lim_{N \rightarrow \infty} \frac{\mathbb{E}^A [N(N+1)(\hat{v} - \hat{v}_{map} + \hat{v}_{map} - \lambda_1 \tilde{y}_1)\hat{\theta}_{map} - N^2\hat{\theta}_{map}^2]}{4(N+1)^2 \lambda_2} \\
&= \frac{\mathbb{E}^A [(\hat{v} - \hat{v}_{map})\hat{\theta}_{map}]}{4\lambda_2} > 0, \tag{23}
\end{aligned}$$

where in the above derivation we have used Eq. (18) and $\mathbb{E}^A[\tilde{v}] = \mathbb{E}^A[\hat{v}(\tilde{y}_1)]$. The expression of the MAP estimate $\hat{\theta}_{map} \equiv (\hat{v}_{map} - \lambda_1 \tilde{y}_1) \mathbf{1}_{|\tilde{y}_1| > K^*}$ implies $(\hat{v}_{map} - \lambda_1 \tilde{y}_1)\hat{\theta}_{map} = \hat{\theta}_{map}^2$. The last expression is strictly positive because $(\hat{v} - \hat{v}_{map})$ and $\hat{\theta}_{map}$ has the same sign for $|\tilde{y}_1| > K^*$. \square

Fig. 4(a) shows the total profit of a hundred arbitrageurs who follow the robust strategy, conditional on the observed order flow y_1 . This profit profile (red curve) is proportional to the term $(\hat{v} - \hat{v}_{map}) \cdot \hat{\theta}_{map}$ in Eq. (23). It exhibits two spikes of profits beyond the trading thresholds (labeled by blue circles). These spikes indicate the major source of their extra

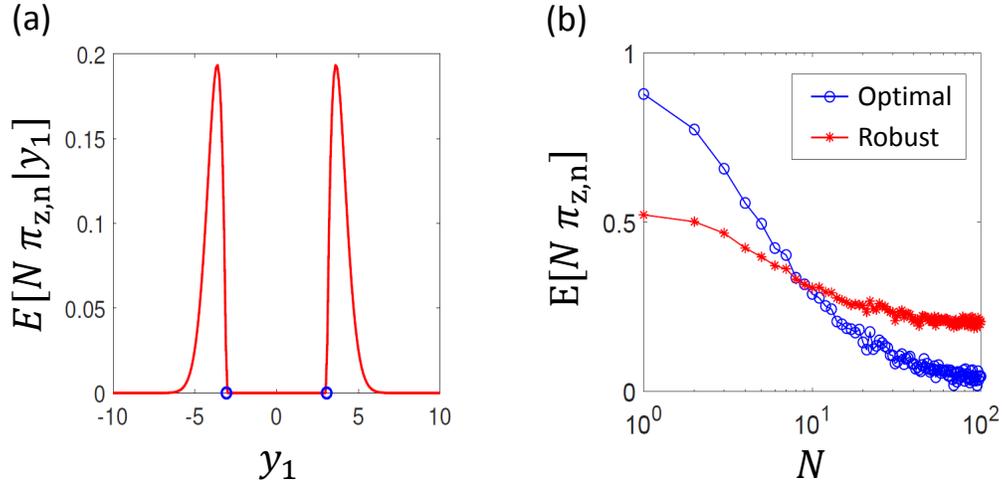


Figure 4. (a) The arbitrageurs’ total profit under the robust strategy conditional on y_1 . (b) The total arbitrage profit under the REE strategy vs. that under the robust strategy.

profits. Intuitively, arbitrageurs’ under-trading is most prominent near the “kinks” of their robust strategy. Their non-competitive profits must be strongest there.

Fig. 4(b) compares the total payoffs to arbitrageurs when they follow different types of strategies. In the oligopolistic case (i.e., small N), the REE strategy allows them to earn higher profits, because the robust strategy ignores a wide range of profit opportunities. As N increases, the profitability of the REE strategy decays faster. In the competitive limit, arbitrageurs compete away their profits under REE and restore market efficiency at $t = 2$.

In contrast, arbitrageurs’ total payoff converges to a positive value when they follow the robust strategy [Fig. 4(b)]. This confirms Proposition 2.5 and indicates a non-competitive effect. Their positive limiting payoff is attributable to the market power they amass beyond the inaction zone, where they trade less aggressively than they would do under REE [Fig. 3 and Fig. 4(a)]. This collusive behavior does not involve any communication device or explicit agreement. Their tacit collusion is not a result of financial constraints or trading frictions. It is due to traders’ robust control for (non-Gaussian) model risk. Outside their inaction region, the cartel effect will prevent the market from being fully efficient .

Corollary 2.5. *In the limit $N \rightarrow \infty$, arbitrageurs will restore market efficiency when they follow the REE strategy, i.e., $\lim_{N \rightarrow \infty} E^A[P_2(\tilde{y}_1, \tilde{y}_2) | \tilde{y}_1] = E^A[\tilde{v} | \tilde{y}_1]$ under $Z_{2,n}^o(s, y_1; \xi_v)$ for $n = 1, \dots, N$; however, market efficiency is hindered when a finite fraction of arbitrageurs follow the robust strategy, i.e., $\lim_{N \rightarrow \infty} E^A[P_2(\tilde{y}_1, \tilde{y}_2) | \tilde{y}_1] \neq E^A[\tilde{v} | \tilde{y}_1]$ under $Z_{2,n}(s, y_1; K^*)$.*

Proof. See Appendix A.5. □

By Corollary 2.5, it is difficult to restore market efficiency even if the economy hosts an infinite number of risk-neutral arbitrageurs. To restore price efficiency in the second period, it requires that (almost) every arbitrageur follows the REE strategy, that is, (almost) every arbitrageur knows *on average* the correct fat-tail prior and has no aversion to uncertainty. This is practically impossible because real-life arbitrageurs face different levels of model risks. Moreover, there exist both internal and external pressures that force them to manage such risks. Their robust control easily translates to their ambiguity aversion, which significantly limits their *willingness* to eliminate mispricings. As reviewed in Gromb and Vayanos (2010), existing studies mostly focus on different costs that limits arbitrageurs’ *ability* in trading. Those frictions could be eased by injecting sufficient capital or removing certain constraints. The mechanism here is different. First, model risk is an intrinsic problem which may not be resolved easily. Second, arbitrageurs here are *able* to eliminate pricing errors; they hesitate to do so because of their aversion to uncertainty²⁴. Third, arbitrageurs’ hesitation in arbitrage has two characteristics: (1) the large inaction region tells them to leave money on the table; (2) their undertrading beyond the inaction region supports them as a “cartel”. Consequently, even with an infinite number of risk-neutral arbitrageurs, a wide range of pricing errors can persist in this economy. This is an endogenous outcome of model risk.

Nowadays, financial markets have been largely occupied by algorithmic traders. The surge of quantitative modeling and machine-learning techniques can bring about hidden issues. The present paper demonstrates that statistical arbitrageurs can use machine-learning tools to combat model uncertainty and similar algorithmic “kinks” in their strategy can mitigate their competition at the expense of market efficiency. This is a general implication, given that many machine-learning algorithms have inaction regions and decision “kinks”.

Equilibrium Condition. In the liquidity regime $\mu < 0$, an arbitrageur may find it profitable to trade in the first period and take advantage of the aggressive feedback trading of other arbitrageurs. One can verify Eq. (6) to see whether this unilateral deviation is profitable.

Corollary 2.6. *The conjectured equilibrium strategy profile may fail in the liquidity regime $\mu < \mu^*(N)$, where $\mu^*(N)$ is the largest root that solves $1 + \frac{N-1}{N+1} \cdot \frac{2}{1+\mu} = \frac{4}{\sqrt{1-\mu}}$. Given a large number of arbitrageurs using the same robust strategy, it can be profitable for an individual trader to deviate from the conjectured no-trade strategy in the first period. This deviation involves trading a large quantity $z_1 \gg K^*$ to trigger the other arbitrageurs and then unwinding the position at more favorable prices supported by the over-aggressive trading of others.*

Proof. See Appendix A.6 □

²⁴Arbitrageurs are risk-neutral but ambiguity-averse in this setup. Their hesitation to perform arbitrage trading is not due to their risk aversion.

3 Model of Savvy Informed Trader

In this section, I extend the previous model to investigate how strategic interaction between the informed trader and the arbitrageurs affect equilibrium outcomes. This model extension can be interpreted as an institutional informed trader optimizes the dynamic order-execution algorithm by taking in account the responses of algorithmic arbitrageurs who use simple machine-learning strategies to exploit her trades. The extended model can be used, for example, to analyze controversial issues in algorithmic trading. It can shed light on hidden risks when algorithmic traders pervade financial markets. Such risks may account for market vulnerability and deserve more attention from regulators.

Let us consider a *savvy informed trader* who observes simultaneously the asset value \tilde{v} and the distribution-type signal \tilde{s} at the beginning. She anticipates the momentum trading of arbitrageurs and behaves strategically. In the Laplacian case, she will consider how her initial trading affects arbitrageurs' next responses. By backward induction, her expected total profit contains a nonlinear term reflecting her consideration of arbitrageurs' nonlinear inference. As a result, her first-period trading strategy is no longer linear and the *rational-expectations equilibrium* (REE) becomes intractable; more discussions are available in Appendix A.7.

To gain insights, the analysis in this section is devoted to a tractable model where strategic arbitrageurs only consider linear-triggering strategies that converge to the REE. This model keeps the basic structure (Table 1) elaborated in the previous section. I present a set of new assumptions to clarify traders' belief systems and information sets.

Assumption 3.1. *As common knowledge, this market has fixed linear pricing schedules, $\tilde{p}_1 = P_1(\tilde{y}_1) = \lambda_1 \tilde{y}_1$ and $\tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2) = \lambda_1 \tilde{y}_1 + \lambda_2 \tilde{y}_2$, that are exogenously given by Eq. (9).*

Assumption 3.2. *Arbitrageurs observe \tilde{s} and have the correct priors: $\mathcal{N}(0, \sigma_v^2)$ at $\tilde{s} = 0$ and $\mathcal{L}(0, \xi_v)$ at $\tilde{s} = 1$. For simplicity, arbitrageurs only consider linear-triggering strategies of the form²⁵: $Z_{2,n}(s = 1, y_1; K_n) = Z^\infty(y_1, \xi_v) \mathbf{1}_{|y_1| > K_n}$, where Z^∞ denotes the asymptotes of their REE strategy to be determined in the limit REE. Each arbitrageur chooses the optimal threshold, taking as given the best responses of other arbitrageurs and the informed trader.*

Assumption 3.3. *The risk-neutral informed trader observes both \tilde{v} and \tilde{s} at $t = 0$. This fact and Assumption 3.3 are held as common knowledge among the informed trader and arbitrageurs. In other words, the informed trader knows everything known by the arbitrageurs, including their prior belief and their adherence to linear-triggering strategies. Arbitrageurs also know everything known by the informed trader except the private information \tilde{v} .*

²⁵Using linear-triggering strategies, arbitrageurs implicitly conjecture that the informed trader's strategy increases with her private signal. However, the Bayesian-rational strategy is not necessarily monotone.

The above assumptions put our focus on the strategic interplay between informed trader and arbitrageurs. The linear pricing rule in Assumption 3.1 can hold when market makers believe that they are living in the two-period Kyle model with the Gaussian prior $\tilde{v} \sim \mathcal{N}(0, \sigma_v^2)$. Arbitrageurs' adherence to linear-triggering strategies in Assumption 3.2 is motivated by the robust strategy discovered in Section 2. If traders worry about the complexity or overtrading of the REE strategy, they may favor such simple algorithms. The suggested linear-triggering strategies are determined by three parameters: slope, intercept, and threshold. These provide well-defined trading rules amenable for computerized executions. Assumption 3.3 explains the "savviness" of this informed trader who is Bayesian-rational, has correct knowledge about the information structure, and anticipates the strategy space of arbitrageurs.

The timeline of this model is identical to Table 1, except that the informed trader observes both \tilde{v} and \tilde{s} at $t = 0$. The strategies of informed trader and arbitrageurs are denoted by $\mathbf{X} = \langle X_1, X_2 \rangle$ and $\mathbf{Z} = [\mathbf{Z}_1, \dots, \mathbf{Z}_N]$, where $\mathbf{Z}_n = \langle Z_{1,n}, Z_{2,n} \rangle$ is the n -th arbitrageur's strategy for $n = 1, \dots, N$. The informed trader knows $\mathcal{I}_{1,x} = \{\tilde{v}, \tilde{s}\}$ before trading at $t = 1$ and $\mathcal{I}_{2,x} = \{\tilde{v}, \tilde{s}, \tilde{y}_1\}$ before trading at $t = 2$. We can write $\tilde{x}_1 = X_1(\tilde{v}, \tilde{s})$ and $\tilde{x}_2 = X_2(\tilde{v}, \tilde{s}, \tilde{y}_1)$. Given the information sets of arbitrageurs, $\mathcal{I}_{1,z} = \{\tilde{s}\}$ and $\mathcal{I}_{2,z} = \{\tilde{s}, \tilde{y}_1\}$, it is justified to write $\tilde{z}_{1,n} = Z_{1,n}(\tilde{s})$ and $\tilde{z}_{2,n} = Z_{2,n}(\tilde{s}, \tilde{y}_1)$ for $n = 1, \dots, N$. Let $\tilde{\pi}_x = \sum_{t=1}^2 (\tilde{v} - \tilde{p}_t) \tilde{x}_t$ be the informed trader's profit, and $\tilde{\pi}_{z,n} = \sum_{t=1}^2 (\tilde{v} - \tilde{p}_t) \tilde{z}_{t,n}$ be the n -th arbitrageur's profit. It is common knowledge that the market-clearing prices are

$$\tilde{p}_1 = P_1(\tilde{y}_1) = \lambda_1 \tilde{y}_1 = \lambda_1 \left(X_1(\tilde{s}, \tilde{v}) + \sum_{n=1}^N Z_{1,n}(\tilde{s}) + \tilde{u}_1 \right), \quad (24)$$

$$\tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2) = \tilde{p}_1 + \lambda_2 \tilde{y}_2 = \lambda_1 \tilde{y}_1 + \lambda_2 \left(X_2(\tilde{s}, \tilde{v}, \tilde{y}_1) + \sum_{n=1}^N Z_{2,n}(\tilde{s}, \tilde{y}_1) + \tilde{u}_2 \right). \quad (25)$$

To stress the dependence of prices on the strategies of traders, we write $\tilde{p}_t = \tilde{p}_t(\mathbf{X}, \mathbf{Z})$ for $t = 1, 2$. We also write $\tilde{\pi}_x = \tilde{\pi}_x(\mathbf{X}, \mathbf{Z})$ and $\tilde{\pi}_{z,n} = \tilde{\pi}_{z,n}(\mathbf{X}, \mathbf{Z})$ because the strategy of informed trader will affect the trading profits of arbitrageurs through direct competition and learning interference, and arbitrageurs' strategies also affect the informed trader's profits through competition and strategic interaction.

In this model, the informed trader and arbitrageurs have the same (consistent) belief system. In particular, they have correct common knowledge about the mixture distribution of \tilde{v} . Since \tilde{s} is observed by all of them at $t = 0$, the informed trader is aware of the time at which arbitrageurs may trade. However, the informed trader cannot fool arbitrageurs into believing a different type of \tilde{v} . It is also common knowledge among them that every arbitrageur adheres to the linear-triggering strategy with only one choice variable: the trading threshold.

Definition of Equilibrium. The equilibrium here is defined as a pair of strategies (\mathbf{X}, \mathbf{Z}) such that, under the market-clearing prices Eq. (24) and Eq. (25), the following conditions hold:

1. For any alternative strategy $\mathbf{X}' = \langle X'_1, X'_2 \rangle$ differing from $\mathbf{X} = \langle X_1, X_2 \rangle$, the strategy \mathbf{X} yields an expected total profit no less than \mathbf{X}' , and also X_2 yields an expected profit in the second period no less than any single deviation X'_2 :

$$\mathbb{E}[\tilde{\pi}_x(\mathbf{X}, \mathbf{Z})|\tilde{v}, \tilde{s}] \geq \mathbb{E}[\tilde{\pi}_x(\mathbf{X}', \mathbf{Z})|\tilde{v}, \tilde{s}], \quad (26)$$

$$\mathbb{E}[(\tilde{v} - \tilde{p}_2(\langle X_1, X_2 \rangle, \mathbf{Z}))X_2|\tilde{v}, \tilde{s}, \tilde{y}_1] \geq \mathbb{E}[(\tilde{v} - \tilde{p}_2(\langle X_1, X'_2 \rangle, \mathbf{Z}))X'_2|\tilde{v}, \tilde{s}, \tilde{y}_1] \quad (27)$$

2. For all $n = 1, \dots, N$ and for any alternative strategy profile \mathbf{Z}' differing from \mathbf{Z} only in the n -th component $\mathbf{Z}'_n = \langle Z'_{1,n}, Z'_{2,n} \rangle$, the strategy \mathbf{Z} yields an expected profit no less than \mathbf{Z}' , and also $Z_{2,n}$ yields an expected profit in the second period no less than $Z'_{2,n}$:

$$\mathbb{E}[\tilde{\pi}_{z,n}(\mathbf{X}, \mathbf{Z})|\tilde{s}] \geq \mathbb{E}[\tilde{\pi}_{z,n}(\mathbf{X}, \mathbf{Z}')|\tilde{s}], \quad (28)$$

$$\mathbb{E}[(\tilde{v} - \tilde{p}_2(\cdot, Z_{2,n}))Z_{2,n}|\tilde{s}, \tilde{y}_1] \geq \mathbb{E}[(\tilde{v} - \tilde{p}_2(\cdot, Z'_{2,n}))Z'_{2,n}|\tilde{s}, \tilde{y}_1]. \quad (29)$$

The strategy profile on the right hand side of Eq. (29) only differs from (\mathbf{X}, \mathbf{Z}) at $Z_{2,n}$.

In the Gaussian case, the informed trader's strategy remains the same as those in Proposition 2.1; arbitrageurs find no trading opportunity in this efficient market. To solve the equilibrium in the fat-tail case, it is useful to conjecture first and verify later that arbitrageurs will not trade in the first period. We first solve their second-period optimal strategy under this no-trade conjecture and then check if it is indeed unprofitable for any arbitrageur to trade in the first period. There is another implicit conjecture in the model development. To follow the linear-triggering strategies, arbitrageurs think that the informed trader plays a monotone strategy which increases with her private signal. This needs to be verified too.

3.1 Equilibrium with Linear-Triggering Strategies

In the fat-tail case, large order flows at $t = 1$ are mostly attributable to the informed trading. This simplifies the inference problem for arbitrageurs as they can conjecture that

$$X_1(s = 1, v) \rightarrow \frac{v}{\rho\lambda_1} + \text{sign}(v)c\kappa\sigma_u, \quad (30)$$

where ρ and c are parameters to be determined in the limit equilibrium. The intercept term, $c\kappa\sigma_u$, reflects how the informed trader exploits her opponents' learning bias, $\kappa\sigma_u = \frac{\rho\lambda_1\sigma_u^2}{\xi_v}$. If

Eq. (30) holds, the arbitrageurs' estimate of \tilde{v} will be asymptotically linear with the past order flow. In Appendix A.8, I solve the asymptotic $X_1(s = 1, v)$ and derive two algebraic equations for ρ and c . Their solutions are given by

$$\rho(\mu, N) = \frac{2 + 5N + N^2 + 2\mu - N\mu - (N + 2)\sqrt{N^2 + (1 + \mu)^2 + 2N(3\mu - 1)}}{2N(1 - \mu)}, \quad (31)$$

$$c(\mu, N) = -\frac{3 + N - \mu - \sqrt{N^2 + (1 + \mu)^2 + 2N(3\mu - 1)}}{1 + N + \mu + \sqrt{N^2 + (1 + \mu)^2 + 2N(3\mu - 1)}} \cdot \frac{N}{2}. \quad (32)$$

Here, the parameter ρ decreases with μ and N , because poorer liquidity or higher competitive pressure tomorrow can stimulate more aggressive informed trading today. The parameter c increases (with μ) from -1 to 0 , because poor future liquidity tends to discourage strategic actions; as shown in Appendix A.9, this parameter reflects the extent of how the informed trader strategically exploits the estimation bias of arbitrageurs. These two parameters can determine the REE asymptotes, Z^∞ , which helps us to pin down the following equilibrium.

Proposition 3.1. *In the liquidity regime of $\mu > \mu_\epsilon$ where $\mu_\epsilon \approx 0.005$ according to numerical results, the following equilibrium (\mathbf{X}, \mathbf{Z}) holds. First, arbitrageurs do not trade in the first period, i.e., $Z_{1,n} = 0$ for $n = 1, \dots, N$. Their optimal linear-triggering strategy at $t = 2$ is*

$$Z_{2,n}(s, y_1; K^*) = sZ^\infty(y_1, \xi_v)\mathbf{1}_{|y_1| > K^*} = s\frac{(1 - \mu)(\rho - 1)}{N + 2} \left[y_1 - \text{sign}(y_1)\frac{\rho(1 + c)\kappa\sigma_u}{\rho - 1} \right] \mathbf{1}_{|y_1| > K^*}, \quad (33)$$

$$K^*(\mu, N) = \max \left[\kappa\sigma_u, \frac{\rho(1 + c)\kappa\sigma_u}{\rho - 1} \right] = \sigma_u \frac{2\sqrt{1 + \mu}}{3 + \mu} \max \left[\rho, \frac{\rho^2(1 + c)}{\rho - 1} \right]. \quad (34)$$

For the informed trader, the equilibrium strategy at $t = 2$ is to trade

$$X_2(v, s, y_1; K^*) = (1 - \mu)\frac{v - \lambda_1 y_1}{2\lambda_1} - s\frac{NZ^\infty(y_1)\mathbf{1}_{|y_1| > K^*}}{2}. \quad (35)$$

The strategy at $t = 1$ is monotone with her signal and solved by Eq. (126) in Appendix A.10.

Proof. See Appendix A.10. □

Corollary 3.1. *The linear-triggering strategy Eq. (33) implies the heuristic learning rule, $\hat{\theta}_T = s \cdot (\hat{v}_T - \lambda_1 y_1)\mathbf{1}_{|y_1| > K^*}$, which estimates $\tilde{\theta} = \tilde{v} - p_1$, with*

$$\hat{v}_T(y_1; \xi_v) = \rho\lambda[y_1 - \text{sign}(y_1)(1 + c)\kappa\sigma_u]\mathbf{1}_{|y_1| > \kappa\sigma_u}. \quad (36)$$

Proof. See Appendix A.10 as well. □

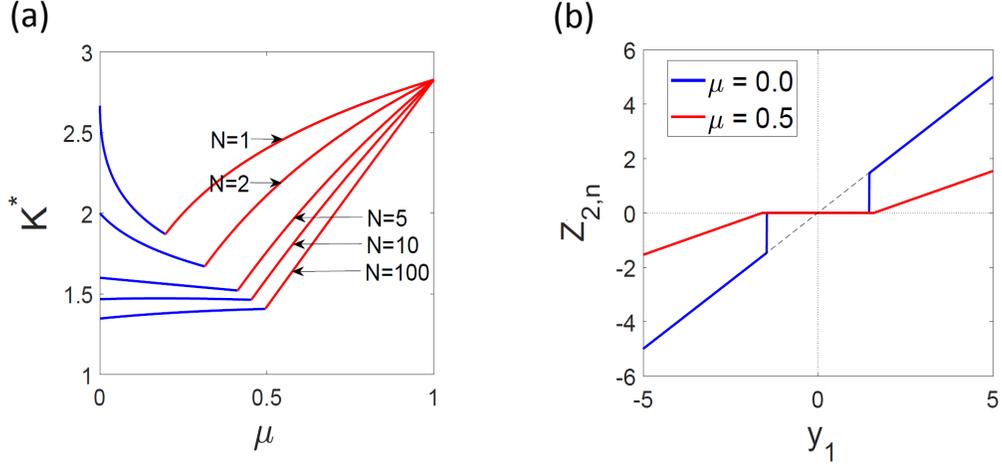


Figure 5. The threshold $K^*(\mu, N)$ and the strategy $Z_{2,n}(s, y_1; K^*)$ in two liquidity regimes.

The learning rule \hat{v}_T looks similar to the MAP estimator \hat{v}_{map} in Eq. (19), except that the horizontal intercept differs by a factor $(1 + c)$. The learning threshold, $\kappa\sigma_u \equiv \frac{\rho\lambda_1\sigma_u^2}{\xi_v}$, is independent of the parameter c , because parallel shifts of the informed trading strategy do not change the signal-to-noise ratio perceived by arbitrageurs. This learning threshold depends on the parameter ρ , because more aggressive informed trading (smaller ρ) can make arbitrageurs learn faster (smaller $\kappa\sigma_u$). The overall learning rule, $\hat{\theta}_T(y_1; K^*)$, is governed by the threshold K^* , which is the maximum of learning threshold $\kappa\sigma_u$ and strategic intercept term $\frac{\rho(1+c)\kappa\sigma_u}{\rho-1}$. Since this intercept increases (with μ) from 0 to $2\kappa\sigma_u$, it must cross $\kappa\sigma_u$ at some intermediate value of μ . This indicates a kink in the equilibrium threshold:

Corollary 3.2. *There are two liquidity regimes separated by the critical liquidity value*

$$\mu_c(N) = \sqrt{N(N+2)^3 - N(N+3)} - 1 \in \left[3\sqrt{3} - 5, \frac{1}{2} \right]. \quad (37)$$

For $\mu \in [0, \mu_c]$, $Z_{2,n}(s, y_1; K^*)$ is discontinuous at $|y_1| = K^* = \kappa\sigma_u$ which decreases with μ . For $\mu \in [\mu_c, 1]$, $Z_{2,n}(s, y_1; K^*)$ is continuous and has $K^* = \frac{\rho(1+c)}{\rho-1}\kappa\sigma_u$ which increases with μ .

Proof. The critical liquidity μ_c is set by the crossover condition $1 = \frac{\rho(1+c)}{\rho-1}$ or $1 + \rho c = 0$. \square

The rescaled threshold K^*/σ_u only depends on the liquidity level μ and the competition condition N (Fig. 5). Under good liquidity $\mu \in [0, \mu_c]$, the equilibrium threshold is set by the learning hurdle of \hat{v}_T , i.e., $K^* = \kappa\sigma_u$. Traders who use a threshold lower than $\kappa\sigma_u$ may engage in unjustified trading for a range of states where their estimated signal \hat{v}_T is zero.

Under poor liquidity $\mu \in [\mu_c, 1]$, the equilibrium threshold is set by the horizontal intercept of $\hat{\theta}_T$, i.e., $K^* = \frac{\rho(1+c)}{\rho-1} \kappa \sigma_u$. Traders who use a threshold lower than this may do contrarian trading for a range of states where their estimated residual signal $\hat{\theta}_T$ is zero. Arbitrageurs will keep undercutting their thresholds as far as possible²⁶ until they hit the lower bound K^* in Eq. (34) which excludes contrarian trading or any unjustified trading.

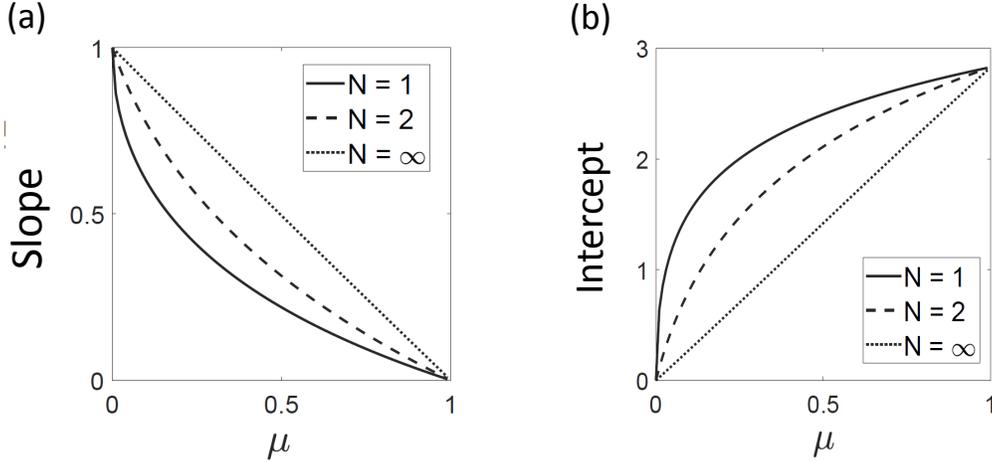


Figure 6. The slope and intercept of $Z_2(y_1) = \sum_{n=1}^N Z_{2,n}(s=1, y_1; K^*)$ as a function of μ .

As shown in Fig. 6, the total arbitrage trading $Z_2(y_1) \equiv \sum_{n=1}^N Z_{2,n}(s=1, y_1; K^*)$ has a slope, $\frac{N(1-\mu)(\rho-1)}{N+2}$, which decreases from 1 to 0 as μ varies from 0 to 1. Its horizontal intercept, $\frac{\rho(1+c)}{\rho-1} \kappa \sigma_u$, increases from 0 to $2\sqrt{2}\sigma_u$. At constant market depth, the total arbitrage trading collapses to the 45° line, $\lim_{\mu \rightarrow 0} Z_2 = y_1 \mathbf{1}_{|y_1| > \kappa \sigma_u}$, regardless of the number N . This is an “order-flow mimicking” strategy, since the total quantity traded by arbitrageurs exactly mimics the total order flow they observed earlier. Also, this is like a pool of stop-loss orders which get triggered to execute whenever the price change surpasses $\lambda_1 \kappa \sigma_u = \frac{4\sqrt{2}}{9} \frac{N+1}{N} \sigma_v$ in either direction. A function of the form, $F(y) = y \mathbf{1}_{|y| > K}$, is often called “hard-thresholding” in machine learning. For $\mu > 0.5$, arbitrageurs always use the “soft-thresholding” strategy.

Let’s look at the strategy of informed trader in different liquidity regimes. If market liquidity at $t=2$ is good ($\mu < \mu_c$), her initial strategy $X_1(s=1, v; K^*)$ is bended toward K^* to distort arbitrageurs’ learning [Fig. 7(a)]. With $\tilde{x}_1 \approx K^*$ for a range of \tilde{v} , it will be difficult for arbitrageurs to infer the strength of \tilde{v} from $\tilde{y}_1 = \tilde{x}_1 + \tilde{u}_1$. Their trading decisions are error-prone because they are largely influenced by noise trading \tilde{u}_1 . The nonlinear pure

²⁶As long as the informed trader’s strategy monotonically increases with her signal, it will be profitable for arbitrageurs to undercut the threshold as much as possible.

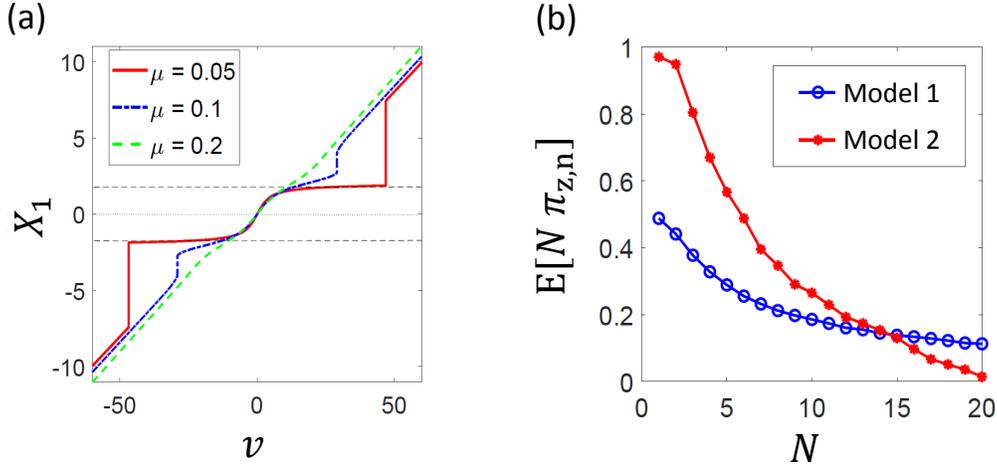


Figure 7. (a) the informed trader's strategy $X_1(s = 1, v)$ under different μ . (b) the total payoffs to arbitrageurs in two models under respective linear-triggering strategies.

strategy allows the informed trader to hide her signal temporarily and inhibit the response of arbitrageurs. If future liquidity is poor ($\mu > \mu_c$), the informed trader will trade more at $t = 1$ and play the game more honestly. Poor liquidity discourages arbitrage trading and reduces the incentive to distort their learning. Overall, the informed trader induces arbitrageurs to trade more competitively. This disrupts their market power and the cartel effect identified in the model of robust arbitrageurs. Facing the savvy informed trader, arbitrageurs can no longer sustain extra market power nor earn noncompetitive profits at large N [Fig. 7(b)].

3.2 Disruptive Strategies and Price Manipulations

In this trading game with linear-triggering strategies, there is an implicit belief in the arbitrageurs' minds that the informed trader will play a monotone strategy which increases with her signal. Numerically, this conjecture is found to hold in the liquidity regime where $\mu > \mu_c \approx 0.005$. However, the conjectured equilibrium becomes unstable when market depth is almost constant ($\mu \rightarrow 0$). If μ is arbitrarily close to 0, the total order flow from arbitrageurs will closely mimic the order flow y_1 . This may invite the informed trader to trick them.

Corollary 3.3. *At $v = 0$ and as $\mu \rightarrow 0$, the informed trader will first trade a sufficiently large x_1 to trigger arbitrageurs and then trade $x_2 = -y_1$ to offset their momentum trading, i.e., $\lim_{\mu \rightarrow 0} X_2(v = 0, y_1) = -y_1 = -\lim_{\mu \rightarrow 0} Z_2(y_1)$. This Bayesian-rational strategy has a terminal position of $x_1 + x_2 = -u_1$ which is zero on average, with an expected profit of $\lambda_1 \sigma_u^2$.*

Proof. This rational strategy follows from Eq. (33) and Eq. (35) by taking both limits $v \rightarrow 0$ and $\mu \rightarrow 0$. Detailed proof can be found in the Appendix A.11. \square

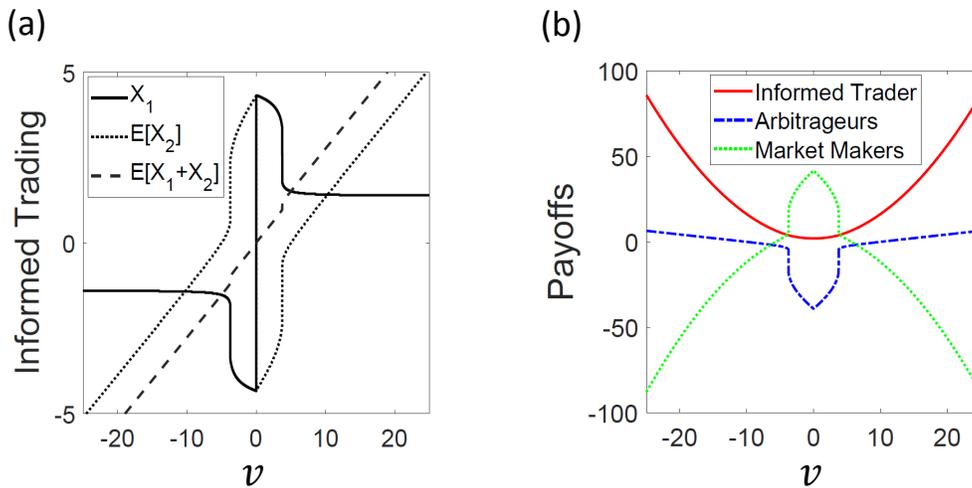


Figure 8. (a) the optimal strategy of informed trader under $\mu = 10^{-4}$, $N = 3$ and $\xi_v = 3$. (b) the total payoffs to different groups of traders.

As shown in Fig. 8(a), when the private signal v is very small, the informed trader places a large order $|x_1| \gg K^* = \kappa\sigma_u$ to trigger arbitrageurs whose trading at $t = 2$ closely mimics the total order flow observed at $t = 1$. This allows the informed trader to liquidate most of her inventory at more favorable prices at $t = 2$. The terminal position $E[\tilde{x}_1 + \tilde{x}_2 | \tilde{v} = v]$ is almost linear with her private signal v , but her strategy in each period is non-monotone with her signal. Fig. 8(b) shows the total payoffs to different groups of traders. Arbitrageurs incur dramatic losses near the origin as they have been fooled by the informed trader who earns a small profit on average. The losses of arbitrageurs mostly benefit market makers.

The non-monotone strategy seems disruptive and resembles controversial strategies in the real world, including *momentum ignition* and *stop-loss hunting*. These schemes are usually regarded as *trade-based price manipulations* by regulators. If such non-monotone strategies are prohibited (by regulators) in the model, the informed trader at the state $v = 0$ will not trade at $t = 1$. Instead, she will watch the market first and trade at $t = 2$ against either the noise-driven price changes or the order flows from arbitrageurs who are falsely triggered.

Kyle and Viswanathan (2008) recommend two economic criteria for regulators to define illegal price manipulations. These are pricing accuracy and market liquidity. Fig. 9 compares the (unconditional) probability distributions of prices when the non-monotone strategy is

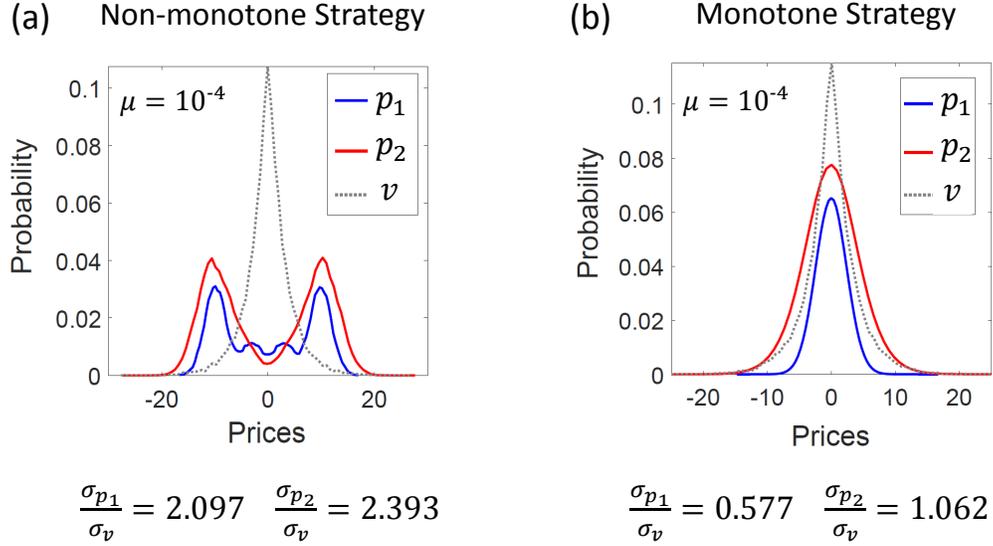


Figure 9. The unconditional probability distributions of the prices \tilde{p}_1 and \tilde{p}_2 under the non-monotone strategy versus the monotone strategy, given $\mu = 10^{-4}$, $N = 3$ and $\sigma_v = 3\sqrt{2}$.

allowed or banned. With the non-monotone strategy in Fig. 8(a), price distributions are bimodal in both periods [Fig. 9(a)]. Pricing accuracy is poor as prices do not reflect the fundamental value \tilde{v} (with a unimodal distribution). Price volatilities are at least twice as large as the fundamental volatility σ_v . If a common investor arrives and trades this asset, she is likely to buy at a much higher ask price or sell at a much lower bid price. The bimodal price pattern reflects a much wider bid-ask spread for common investors. In contrast, if regulators set rules to ban such disruptive strategies, the price distributions become bell-shaped in both periods with reasonable price volatilities and pricing accuracy [Fig. 9(b)].

Regulators need to sort out the economic conditions for the *trade-base manipulations*. The results in this paper prescribe a list of conditions that could be necessary for the non-monotone disruptive strategy.

- (1) Speculators think that market makers set inaccurate prices by using incorrect priors.
- (2) Speculators have fat-tail priors about the fundamental value or trading opportunities.
- (3) There is strategic interplay between the informed trader and those speculators.
- (4) Market depth is not decreasing when the informed trader liquidates her inventory.
- (5) Traders face no trading costs, no inventory costs, nor threat from regulators.
- (6) There is no other informed trader who could interfere with the disruptive strategy.

The (non-monotone) disruptive strategy may fail if any of these conditions is not satisfied. It seems not easy at all, but the key condition is that the total feedback trading from

speculators has a slope no less than one. This could happen if speculators underestimate the actual number of speculators (N), since each speculator’s demand is inversely proportional to the number of competitors (estimated by the speculator). This could also happen in the liquidity regime with $\mu < 0$, where the informed trader could dump her early inventory at a lower cost and speculators may trade more aggressively. In the conjectured equilibrium, the response slope of each speculator is given by $\frac{(1-\mu)(\rho-1)}{N+2}$. If all speculators keep using this strategy in the liquidity regime $\mu < 0$, the slope of their aggregate response will be greater than one: $N\frac{(1-\mu)(\rho-1)}{N+2} > 1$. Over-trading makes speculators susceptible to “disruptive attacks”. For the informed trader, the profits of tricking speculators can be outweighed by the losses if she fails to liquidate the undesirable inventory in the second period.

4 Conclusion

This paper studies an equilibrium model of strategic arbitrage in the fat-tail environment. The presence of arbitrageurs is rationalized by applying random fat-tail shocks to the standard Kyle model where market makers adhere to Gaussian beliefs. If arbitrageurs are uncertain about the various of fat-tail shocks, their robust strategy under the max-min choice criteria is operationally equivalent to the LASSO algorithm in machine learning. For robustness, arbitrageurs choose to ignore a wide range of small (uncertain) mispricings and take actions only on large (certain) ones. This strategy is highly effective given its infrequent trading activity. As a result, many anomalies may be detected *ex post* by an external econometrician based on historical data in this economy. The econometrician may conclude that market inefficiency is due to arbitrageurs’ behavioral bias as they overlook those anomalies. In fact, arbitrageurs are rational under their robust-control objective. They use Bayes rule to carefully evaluate all possible states over their multiple priors. Arbitrageurs can amass significant market power due to their under-trading beyond the kinks of robust strategy. This cartel effect allows them to earn noncompetitive profits which do not vanish even if their number goes to infinity. Therefore, price efficiency is further impaired.

If the informed trader strategically interacts with those arbitrageurs, she will try to distort their learning and induce them to trade more aggressively. Under certain market conditions, the informed trader may play a disruptive strategy that resembles real-life controversial practices (like momentum ignition). Such trading schemes can distort the informational content of prices and destabilize stock prices at the expense of common investors.

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A Appendix

A.1 Proof of Proposition 2.1

Under their common belief, the informed trader and market makers first conjecture that arbitrageurs do not trade if the market is efficient. As in the two-period Kyle (1985) model, they can seek a linear equilibrium (\mathbf{X}, \mathbf{P}) , where $\mathbf{P} = \langle P_1, P_2 \rangle$ is the linear pricing strategy of market makers. Let $P_1(y_1) = \lambda_1 y_1$ and $P_2(y_1, y_2) = \lambda_1 y_1 + \lambda_2 y_2$. The information set of informed trader before trading at $t = 2$ is $\mathcal{I}_{2,x} = \{v, y_1\}$. After $t = 1$, she conjectures the price at $t = 2$ as

$$\tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2) = \lambda_1 y_1 + \lambda_2 [X_2(v, y_1) + \tilde{u}_2], \quad \text{under } \{\mathcal{I}_{2,x}, \mathcal{B}\}. \quad (38)$$

Her optimal strategy at $t = 2$ under the information set $\mathcal{I}_{2,x}$ and belief system \mathcal{B} is

$$X_2(v, y_1) = \arg \max_{x_2} \mathbb{E}^{\mathcal{B}} [(v - \tilde{p}_2)x_2 | \mathcal{I}_{2,x}] = \frac{v - \lambda_1 y_1}{2\lambda_2}. \quad (39)$$

The informed trader conjectures the price at $t = 1$ to be $\tilde{p}_1 = \lambda_1 [X_1(v) + \tilde{u}_1]$ under $\{\mathcal{I}_{1,x}, \mathcal{B}\}$. With this notion and $X_2(v, y_1)$, her subjective expected profit is a quadratic function of x_1 :

$$\Pi_x(v, x_1) = x_1(v - \lambda_1 x_1) + \mathbb{E}^{\mathcal{B}} \left[\frac{(v - \lambda_1(x_1 + \tilde{u}_1))^2}{4\lambda_2} \middle| \mathcal{I}_{1,x} = \{v\} \right]. \quad (40)$$

The first order condition is $0 = v - 2\lambda_1 x_1 - \frac{v - \lambda_1 x_1}{2\delta}$, where $\delta \equiv \frac{\lambda_2}{\lambda_1}$. The optimal strategy is

$$X_1(v) = \frac{2\delta - 1}{4\delta - 1} \cdot \frac{v}{\lambda_1} = \frac{v}{\rho\lambda_1}, \quad (41)$$

where $\rho = \frac{4\delta - 1}{2\delta - 1}$. The above results constitute Eq. (11) in Proposition 2.1. Market makers hold the same Gaussian belief. As an extension of Proposition 1 in Huddart et al. (2001), it takes some similar calculations to derive that $\lambda_1 = \frac{\sqrt{2\delta(2\delta - 1)}}{4\delta - 1} \frac{\sigma_v}{\sigma_u}$, where the ratio δ is given by the largest root to the cubic equation:

$$8\gamma\delta^3 - 4\gamma\delta^2 - 4\delta + 1 = 0. \quad (42)$$

Here, $\gamma > 0$ is the ratio of noise trading volatilities over time. Under this pair of linear strategies \mathbf{X} and \mathbf{P} , prices are conditional expectations of public information under market makers' belief \mathcal{B} . So the informed trader and market makers believe that if they play \mathbf{X} and \mathbf{P} no arbitrageurs would trade. This confirms the initial conjecture and completes the proof.

A.2 Proof of Proposition 2.2

Arbitrageurs know that they are not anticipated to trade by the informed trader and market makers. In the Gaussian case ($s = 0$), they have no informational advantage over market makers. The market is efficient under the subgame perfect equilibrium (\mathbf{X}, \mathbf{P}) in Proposition 2.1. Indeed, arbitrageurs will not trade when $s = 0$. In the Laplacian case ($s = 1$), they can exploit the pricing bias because market makers use the wrong prior. To solve the equilibrium, I conjecture first and verify later that arbitrageurs do not trade in the first period, i.e., $Z_{1,n} = 0$ for $n = 1, \dots, N$. Under this conjecture, I solve their optimal strategy at $t = 2$. Arbitrageurs anticipate the informed trader's linear strategy and the market-clearing price,

$$P_2(\tilde{y}_1, \tilde{y}_2) = \lambda_1 \tilde{y}_1 + \lambda_2 \left(X_2(\tilde{v}, \tilde{y}_1) + \sum_{n=1}^N Z_{2,n}(\tilde{s}, \tilde{y}_1) + \tilde{u}_2 \right). \quad (43)$$

They estimate \tilde{v} based on the observed y_1 and their Laplace prior $\mathcal{L}(0, \tilde{\xi})$. In the absence of model risk (i.e., $\tilde{\xi} = \xi$), the n -th arbitrageur solves her optimal strategy,

$$Z_{2,n}^o(s = 1, y_1; \xi) = \arg \max_{z_{2,n}} \mathbb{E}^{\mathcal{A}}[(\tilde{v} - \tilde{p}_2)z_{2,n} | \mathcal{I}_{2,z}], \quad (44)$$

under $\mathcal{I}_{2,z} \equiv \{s, y_1\}$ and the belief $\mathcal{A} = \{s, \xi\}$. Let $Z_{2,-n}^o = \sum_{m \neq n} Z_{2,m}^o$ be the their aggregate trading except the n -th arbitrageur's. The first order condition for $z_{2,n}$ is

$$\mathbb{E}^{\mathcal{A}}[\tilde{v} | \mathcal{I}_{2,z}] - \lambda_1 y_1 = \lambda_2 \left(\mathbb{E}^{\mathcal{A}}[X_2 | \mathcal{I}_{2,z}] + 2z_{2,n} + \mathbb{E}^{\mathcal{A}}[Z_{2,-n}^o | \mathcal{I}_{2,z}] \right). \quad (45)$$

Since $\mathbb{E}^{\mathcal{A}}[X_2 | \mathcal{I}_{2,z}] = \frac{\hat{v} - \lambda_1 y_1}{2\delta\lambda_1}$ where $\hat{v} = \hat{v}(y_1; \xi) = \mathbb{E}^{\mathcal{A}}[\tilde{v} | \mathcal{I}_{2,z}]$, the solution is

$$Z_{2,n}^o(s = 1, y_1; \xi) = \frac{\hat{v} - \lambda_1 y_1}{2\delta\lambda_1} - \frac{\mathbb{E}^{\mathcal{A}}[X_2 | \mathcal{I}_{2,z}] + \mathbb{E}^{\mathcal{A}}[Z_{2,-n}^o | \mathcal{I}_{2,z}]}{2} = \frac{\hat{v} - \lambda_1 y_1}{4\delta\lambda_1} - \frac{\mathbb{E}^{\mathcal{A}}[Z_{2,-n}^o | \mathcal{I}_{2,z}]}{2}. \quad (46)$$

The n -th arbitrageur conjectures that every other arbitrageur solves the same problem and trades $Z_{2,m}^o = \eta \cdot (\hat{v} - p_1)$ for any $m \neq n$, with a coefficient η to be solved. Eq. (46) becomes

$$Z_{2,n}^o(s = 1, y_1; \xi) = \frac{\hat{v} - \lambda_1 y_1}{4\delta\lambda_1} - (N-1) \frac{\eta(\hat{v} - \lambda_1 y_1)}{2} = [\delta^{-1} - 2\lambda_1 \eta(N-1)] \frac{(\hat{v} - \lambda_1 y_1)}{4\lambda_1}. \quad (47)$$

Since each arbitrageur makes the same conjecture in a symmetric equilibrium, they find that $\eta = \frac{\delta^{-1} - 2\lambda_1 \eta(N-1)}{4\lambda_1}$, which has a unique solution

$$\eta = \frac{1}{2\delta\lambda_1(N+1)} > 0. \quad (48)$$

Without model risk, the optimal strategy of arbitrageurs under the Laplace prior $\mathcal{L}(0, \xi)$ is

$$Z_{2,n}^o(s, y_1; \xi) = \frac{\hat{v}(y_1; \xi) - \lambda_1 y_1}{2(N+1)\delta\lambda_1} = \frac{1-\mu}{N+1} \cdot \frac{\hat{\theta}(y_1; \xi)}{2\lambda_1}, \quad n = 1, \dots, N. \quad (49)$$

Since $X_1(v) = \frac{v}{\rho\lambda_1}$, arbitrageurs have a Laplace prior for \tilde{x}_1 , denoted $f_L(x_1) = \frac{\rho\lambda_1}{2\xi} \exp\left(-\frac{\rho\lambda_1|x_1|}{\xi}\right)$. By Bayes' rule, the posterior probability of the informed trading x_1 conditional on y_1 is

$$f(x_1|y_1) = \frac{f(y_1|x_1)f_L(x_1)}{f(y_1)} = \frac{\rho\lambda_1}{2\xi f(y_1)\sqrt{2\pi\sigma_u^2}} \exp\left[-\frac{(y_1-x_1)^2}{2\sigma_u^2} - \frac{\rho\lambda_1|x_1|}{\xi}\right]. \quad (50)$$

The probability density function of $\tilde{y}_1 = \frac{\tilde{v}}{\rho\lambda_1} + \tilde{u}_1$ is found to be:

$$f(y_1) = \frac{\rho\lambda_1}{4\xi} \exp\left(\frac{\rho^2\lambda_1^2\sigma_u^2}{2\xi^2}\right) \left[e^{-\frac{\rho\lambda_1 y_1}{\xi}} \operatorname{erfc}\left(\frac{\rho\lambda_1\sigma_u^2/\xi - y_1}{\sqrt{2}\sigma_u}\right) + e^{\frac{\rho\lambda_1 y_1}{\xi}} \operatorname{erfc}\left(\frac{\rho\lambda_1\sigma_u^2/\xi + y_1}{\sqrt{2}\sigma_u}\right) \right]. \quad (51)$$

I define a dimensionless parameter $\kappa \equiv \frac{\rho\lambda_1\sigma_u}{\xi}$ and rewrite $f(y_1)$ in a dimensionless form

$$f(y_1 = y'\sigma_u) = \frac{\kappa e^{\frac{\kappa^2}{2}}}{4\sigma_u} \left[e^{-\kappa y'} \operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right) + e^{\kappa y'} \operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right) \right], \quad (52)$$

which is symmetric and decays exponentially at large $|y'|$. Bayes' rule implies that

$$\mathbb{E}^A[\tilde{x}_1 = x'\sigma_u | y_1 = y'\sigma_u, \xi] = \sigma_u \int_{-\infty}^{\infty} x f(x|y) dx = \sigma_u \int_{-\infty}^{\infty} \frac{x f(y|x) f(x)}{f(y)} dx, \quad (53)$$

Given that $X_1(v) = \frac{v}{\rho\lambda_1}$, it is easy to derive the posterior expectation of \tilde{v} explicitly:

$$\hat{v} = \mathbb{E}^A[\tilde{v} | y_1 = y'\sigma_u, \xi] = \frac{\kappa\xi(y' - \kappa) \operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right)}{\operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right) + e^{2\kappa y'} \operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right)} + \frac{\kappa\xi(y' + \kappa) \operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right)}{\operatorname{erfc}\left(\frac{\kappa + y'}{\sqrt{2}}\right) + e^{-2\kappa y'} \operatorname{erfc}\left(\frac{\kappa - y'}{\sqrt{2}}\right)}. \quad (54)$$

The rescaled \hat{v}/ξ is an increasing function of y' with a single shape parameter κ . Asymptotic linearity holds at $|y'| \gg \kappa$ that $\hat{v} \rightarrow \rho\lambda_1[y_1 - \operatorname{sign}(y_1)\kappa\sigma_u]$. All the second-order conditions are easy to check. The REE corresponds to the equilibrium where all arbitrageurs have the correct prior. Under REE, we have $\xi = \xi_v = \frac{\sigma_v}{\sqrt{2}}$ such that the shape parameter becomes

$$\kappa(\xi = \xi_v) = \frac{\rho\lambda_1\sigma_u}{\xi_v} = \frac{4\delta - 1}{2\delta - 1} \frac{\sqrt{4\delta(2\delta - 1)}}{4\delta - 1} = \frac{2}{\sqrt{1 + \mu}}, \quad (55)$$

where $\mu \equiv 1 - \frac{1}{\delta}$ quantifies the percentage change of market depth in the second period.

To verify that no arbitrageurs would trade in the first period, I examine the condition Eq. (6). Suppose the n -th arbitrageur deviates from the conjectured strategy by trading a nonzero quantity $Z_{1,n}^{o,d} = z_1 \neq 0$ in the first period. Then the actual total order flow at $t = 1$ is $\tilde{y}'_1 = \tilde{x}_1 + \tilde{z}_1 + \tilde{u}_1$, instead of $\tilde{y}_1 = \tilde{x}_1 + \tilde{u}_1$ in the conjectured equilibrium. Taking \mathbf{X} , \mathbf{P} , and $Z_{2,m}^o(s, y'_1; \xi) = s \frac{\hat{v}(y'_1) - \lambda_1 y'_1}{2(N+1)\delta\lambda_1}$ for any $m \neq n$ as given, the n -th arbitrageur's optimal strategy at $t = 2$ conditional on the information set $\mathcal{I}'_{2,z} = \{s, y_1, z_1\}$ is

$$\begin{aligned}
Z_{2,n}^{o,d}(s, y'_1; \xi) &= s \frac{\hat{v}(y_1; \xi) - \lambda_1 y'_1}{2\delta\lambda_1} - s \frac{\text{E}^A[X_2(\tilde{v}, y'_1)|\mathcal{I}'_{2,z}] + \text{E}^A[Z_{2,-n}^o(s, y'_1; \xi)|\mathcal{I}'_{2,z}]}{2} \\
&= s \frac{\hat{v}(y_1; \xi) - \lambda_1 y'_1}{4\delta\lambda_1} - s \frac{Z_{2,-n}^o(s, y'_1; \xi)}{2} \\
&= s \frac{\hat{v}(y_1; \xi) - \lambda_1 y'_1}{4\delta\lambda_1} - s \frac{(N-1)[\hat{v}(y'_1; \xi) - \lambda_1 y'_1]}{4(N+1)\delta\lambda_1} \\
&= s \frac{\hat{v}(y_1; \xi) - \lambda_1 y'_1}{2(N+1)\lambda_2} + s \frac{N-1}{4(N+1)\lambda_2} [\hat{v}(y_1; \xi) - \hat{v}(y'_1; \xi)]. \tag{56}
\end{aligned}$$

If the n -th trader does not deviate from the no-trade strategy in the first period, her optimal strategy should be $Z_{2,n}^o(s, y_1; \xi) = s \frac{\hat{v}(y_1; \xi) - \lambda_1 y_1}{2(N+1)\delta\lambda_1}$, where $y_1 = x_1 + u_1$. For convenience, we just need to consider the case $s = 1$. Let's add the notation that $\Delta P_1 \equiv \lambda_1(\tilde{y}'_1 - \tilde{y}_1) = \lambda_1 z_1$, $\Delta \hat{v} \equiv \hat{v}(\tilde{y}'_1; \xi) - \hat{v}(\tilde{y}_1; \xi)$, $\Delta Z \equiv Z_{2,n}^{o,d}(s, y'_1; \xi) - Z_{2,n}^o(s, y_1; \xi) = -\frac{\lambda_1 z_1}{2(N+1)\lambda_2} - \frac{N-1}{4(N+1)\lambda_2} \Delta \hat{v}$ and

$$\begin{aligned}
\Delta P_2 &\equiv P_2(\mathbf{X}, \mathbf{Z}') - P_2(\mathbf{X}, \mathbf{Z}) \\
&= \lambda_1 z_1 + \lambda_2 [\Delta Z + X_2(\tilde{v}, \tilde{y}'_1) - X_2(\tilde{v}, \tilde{y}_1) + Z_{2,-n}^o(\tilde{y}'_1) - Z_{2,-n}^o(\tilde{y}_1)] \\
&= \lambda_1 z_1 - \frac{\lambda_1 z_1}{2(N+1)} - \frac{N-1}{4(N+1)} \Delta \hat{v} - \frac{\lambda_1 z_1}{2} + \frac{(N-1)(\Delta \hat{v} - \lambda_1 z_1)}{2(N+1)} \\
&= \frac{\lambda_1 z_1}{2(N+1)} + \frac{N-1}{4(N+1)} \Delta \hat{v} = -\lambda_2 \Delta Z, \tag{57}
\end{aligned}$$

where \mathbf{Z}' differs from $\mathbf{Z} \equiv [\langle 0, Z_{2,1}^o \rangle, \dots, \langle 0, Z_{2,N}^o \rangle]$ only in the n -th element $(\mathbf{Z}')_n = \langle z_1, Z_{2,n}^{o,d} \rangle$. Since $\tilde{y}_1 = X_1(\tilde{v}) + \tilde{u}_1$, we have $\text{E}^A[\tilde{y}_1 \cdot z_1] = 0$ and $\text{E}^A[\hat{v}(\tilde{y}_1) \cdot z_1] = 0$. The payoff difference is

$$\begin{aligned}
\Delta \Pi_{z,n}^d &= \text{E}^A[(\tilde{v} - \tilde{p}_2(\mathbf{X}, \mathbf{Z}')) Z_{2,n}^{o,d} + (\tilde{v} - \tilde{p}_1(\mathbf{X}, \mathbf{Z}')) z_1 - (\tilde{v} - \tilde{p}_2(\mathbf{X}, \mathbf{Z})) Z_{2,n}^o | \tilde{s} = 1, \tilde{\xi} = \xi] \\
&= \text{E}^A[\tilde{v} z_1 - z_1 \tilde{p}_1(\mathbf{X}, \mathbf{Z}') + \tilde{v} \Delta Z - \Delta P_2 \cdot Z_{2,n}^{o,d} - \tilde{p}_2(\mathbf{X}, \mathbf{Z}) \cdot \Delta Z | \tilde{s} = 1, \tilde{\xi} = \xi] \\
&= -\lambda_1 z_1^2 + \text{E}^A[\text{E}^A[(\tilde{v} - \tilde{p}_2(\mathbf{X}, \mathbf{Z}) + \lambda_2 Z_{2,n}^{o,d}) \Delta Z | \tilde{y}_1]] \\
&= -\lambda_1 z_1^2 + \text{E}^A \left[\left(\frac{\hat{v}(\tilde{y}_1; \xi) - \lambda_1 \tilde{y}_1}{N+1} + \lambda_2 \Delta Z \right) \cdot \Delta Z \right] \\
&= -\lambda_1 z_1^2 + \frac{\text{E}^A[(\lambda_1 z_1 + \frac{1}{2}(N-1)\Delta \hat{v})^2]}{4(N+1)^2 \lambda_2} - \frac{N-1}{4(N+1)^2 \lambda_2} \text{E}^A[(\hat{v}(\tilde{y}_1; \xi) - \lambda_1 \tilde{y}_1) \cdot \Delta \hat{v}]. \tag{58}
\end{aligned}$$

One can rewrite Eq. (58) in a symmetric form with respect to z_1 :

$$\begin{aligned}\Delta\Pi_{z,n}^d &= -\lambda_1 z_1^2 + \frac{\mathbb{E}^{\mathcal{A}}[(\lambda_1 z_1)^2 + \frac{1}{4}(N-1)^2(\Delta\hat{v})^2 - (N-1)[\hat{v}(\tilde{y}_1; \xi) - \lambda_1(\tilde{y}_1 + z_1)]\Delta\hat{v}]}{4(N+1)^2\lambda_2} \\ &= -\lambda_1 z_1^2 + \frac{(\lambda_1 z_1)^2 + \mathbb{E}^{\mathcal{A}}[\frac{1}{4}(N-1)^2(\Delta\hat{v})^2 - (N-1)(\hat{\theta}(\tilde{y}_1 + z_1; \xi) - \Delta\hat{v})\Delta\hat{v}]}{4(N+1)^2\lambda_2}.\end{aligned}\quad (59)$$

This is an even function of z_1 because one can use the symmetry of \tilde{y}_1 and $\hat{v}(\cdot)$ to prove

$$\begin{aligned}\mathbb{E}^{\mathcal{A}}[\hat{\theta}(\tilde{y}_1 - z_1; \xi) \cdot \Delta\hat{v}(\tilde{y}_1, -z_1; \xi)] &= \mathbb{E}^{\mathcal{A}}[\hat{\theta}(\tilde{y}_1 - z_1; \xi)(\hat{v}(\tilde{y}_1 - z_1; \xi) - \hat{v}(\tilde{y}_1; \xi))] \\ &= \mathbb{E}^{\mathcal{A}}[\hat{\theta}(-\tilde{y}_1 - z_1; \xi)(\hat{v}(-\tilde{y}_1 - z_1; \xi) - \hat{v}(-\tilde{y}_1; \xi))] \\ &= \mathbb{E}^{\mathcal{A}}[-\hat{\theta}(\tilde{y}_1 + z_1; \xi)(-\hat{v}(\tilde{y}_1 + z_1; \xi) + \hat{v}(\tilde{y}_1; \xi))] = \mathbb{E}^{\mathcal{A}}[\hat{\theta}(\tilde{y}_1 + z_1; \xi) \cdot \Delta\hat{v}(\tilde{y}_1, z_1; \xi)].\end{aligned}$$

The first term of Eq. (59) is the average cost to play z_1 at $t = 1$, whereas the second term represents the average profit from exploiting the biased response of other traders at $t = 2$. The profit of this strategic exploitation has an upper limit which is achieved when all the arbitrageurs have the extreme fat-tail prior $\xi \rightarrow \infty$. In this limit, their response to the past order flow is the strongest and exactly linear with y_1 : $\lim_{\xi \rightarrow \infty} Z_{2,n}^o = \frac{y_1}{(N+1)(2\delta-1)}$. Since $\Delta\Pi_{z,n}^d(-z_1) = \Delta\Pi_{z,n}^d(z_1)$, we only need to consider the positive deviation. For any $z_1 > 0$,

$$\Delta\hat{v}(\tilde{y}_1, z_1; \xi) \equiv \hat{v}(\tilde{y}_1 + z_1; \xi) - \hat{v}(\tilde{y}_1; \xi) \leq \rho\lambda_1(\tilde{y}'_1 - \tilde{y}_1) = \rho\lambda_1 z_1, \quad (60)$$

where the equality holds at $\xi \rightarrow \infty$. Given that $\lim_{\xi \rightarrow \infty} \hat{\theta}(\tilde{y}'_1; \xi) = \lambda_1(\rho - 1)(\tilde{y}_1 + z_1)$, I find

$$\begin{aligned}\Delta\Pi_{z,n}^d &< \lim_{\xi \rightarrow \infty} \Delta\Pi_{z,n}^d = -\lambda_1 z_1^2 + \lambda_1^2 \frac{z_1^2 + \frac{1}{4}(N-1)^2\rho^2 z_1^2 - (N-1)\rho\mathbb{E}^{\mathcal{A}}[((\rho-1)(\tilde{y}_1 + z_1) - \rho z_1)z_1]}{4(N+1)^2\lambda_2} \\ &= -\lambda_1 z_1^2 + (1-\mu)\lambda_1 z_1^2 \frac{[(N-1)\rho + 2]^2}{16(N+1)^2}\end{aligned}\quad (61)$$

The last expression of Eq. (61) is negative for any $\mu > \mu^*(N)$ where $\mu^*(N)$ is the largest root to the equation: $1 + \left(\frac{N-1}{N+1}\right) \frac{2}{1+\mu} = \frac{4}{\sqrt{1-\mu}}$. The maximum of $\mu^*(N)$ is found to be $\mu_\epsilon \equiv \lim_{N \rightarrow \infty} \mu^*(N) \approx -0.23191$, which is the largest root to the cubic equation:

$$\mu^3 + 21\mu^2 + 35\mu + 7 = 0. \quad (62)$$

In the liquidity regime of $\mu > \mu_\epsilon \approx -0.23191$, it is indeed unprofitable for any individual arbitrageur to trade in the first period, i.e., $\Delta\Pi_{z,n}^d(z_1) < 0$ for any $z_1 \neq 0$. This confirms the no-trade conjecture at $t = 1$ and completes the proof of Proposition 2.2.

A.3 Proof of Proposition 2.3

All admissible strategies must lie in the area enclosed by $Z_{2,n}^o(y_1, \xi \rightarrow 0)$, $Z_{2,n}^o(y_1, \xi \rightarrow \infty)$, and the REE asymptotes $Z^\infty(y_1, K^*)$. Any strategy that runs outside this region will violate either the asymptotic requirement or the condition of convexity/concavity preservation. By symmetry, we just discuss the positive domain where the REE strategy is always convex. To satisfy the convexity-preservation rule, the first derivative of an admissible strategy, $\frac{\partial Z'_{2,n}}{\partial y_1}$, can never decrease in the domain of $y_1 > 0$. With a non-decreasing first derivative, the admissible strategy can never go beyond the asymptote $Z^\infty(y_1, K^*)$ and curve back to it.

For $y_1 \in [0, K^*]$, any selling decision located in the bottom triangle “a” would lose money in the worst-case scenario (i.e., if the highest prior ξ_H is true, under which one should buy). Similarly, any buying decision located in the up triangle “b” would lose money in the worst-case scenario (i.e., if the lowest prior ξ_L is true, under which one should buy). This argument indicates a no-trade strategy over $y_1 \in [0, K^*]$. For any $y_1 > K^*$, I will prove that any buying decision $Z'_{2,n}(y_1)$ located inside the area “c” may either lose more money or earn less money than the buying decision $Z^\infty(y_1, K^*)$ determined by the REE asymptotes. Let $Z_\Delta \equiv Z'_{2,n}(y_1) - Z^\infty(y_1, K^*)$. The difference of their payoffs under the lowest prior ξ_L is

$$\begin{aligned} \mathbb{E}^A[\Delta \tilde{\pi}_{z,n} | y_1, \tilde{\xi} = \xi_L] &= \mathbb{E}^A \left[(\tilde{v} - \lambda_1 y_1 - \lambda_2 (X_2 + Z'_{2,n} + Z_{2,-n} + \tilde{u}_2)) Z'_{2,n} \middle| y_1, \tilde{\xi} = \xi_L \right] \\ &\quad - \mathbb{E}^A \left[(\tilde{v} - \lambda_1 y_1 - \lambda_2 (X_2 + Z^\infty + Z_{2,-n} + \tilde{u}_2)) Z^\infty \middle| y_1, \tilde{\xi} = \xi_L \right] \\ &= \mathbb{E}^A \left[Z_\Delta \left[\frac{\tilde{\theta}}{2} - \lambda_2 (Z^\infty + Z_{2,-n} + \tilde{u}_2) \right] \middle| y_1, \tilde{\xi} = \xi_L \right] - \lambda_2 Z'_{2,n} Z_\Delta \end{aligned} \quad (63)$$

The worst-case scenario is that ξ_L is true and every other arbitrageur trades $Z^\infty(y_1, K^*)$. Let $\hat{\theta}_L(y_1; \xi_L) \equiv \mathbb{E}^A[\tilde{\theta} | y_1, \xi_L]$ and $Z^L \equiv \frac{\hat{\theta}_L}{2(N+1)\lambda_2}$. Obviously, $Z^L < Z^\infty < Z'_{2,n}$ and $Z_\Delta > 0$. It is not a profitable deviation for anyone to trade more than $Z^\infty(y_1, K^*)$, since

$$\begin{aligned} \mathbb{E}^A[\Delta \tilde{\pi}_{z,n} | y_1, \tilde{\xi} = \xi_L] &= \lambda_2 Z_\Delta [(N+1)Z^L - Z^\infty - (N-1)Z^\infty] - \lambda_2 Z'_{2,n} Z_\Delta \\ &= \lambda_2 Z_\Delta ((N+1)Z^L - NZ^\infty - Z'_{2,n}) < 0. \end{aligned} \quad (64)$$

So the robust strategy is to follow the REE asymptote, $Z^\infty(y_1, K^*)$, for any $y_1 > K^*$.

By symmetry, the robust strategy is exactly Eq. (17). It remains to verify that no arbitrageur would find it profitable to trade in the first period, given that the other arbitrageurs only trade at $t = 2$ using the same robust strategy. The proof of no-trade condition Eq. (6) will be similar to the proof in Proposition 2.2; see Appendix A.6 for more details.

A.4 Proof of Proposition 2.4 and Corollary 2.4

Under the prior $\mathcal{L}(0, \xi_v)$, the Maximum a Posteriori (MAP) estimate of \tilde{v} given y_1 is

$$\hat{v}_{map} = \arg \max_v f(v|y_1) = \arg \max_v f(y_1|v) f_L(v) = \arg \max_v \exp \left[-\frac{(y_1 - \frac{v}{\rho\lambda_1})^2}{2\sigma_u^2} - \frac{|v|}{\xi_v} \right], \quad (65)$$

We need to find the point of v that minimizes $(y_1 - \frac{v}{\rho\lambda_1})^2 + \frac{2\sigma_u^2|v|}{\xi_v}$ whose first order condition is $y_1 = \frac{v}{\rho\lambda_1} + \kappa\sigma_u \text{sign}(v)$. Graphically inverting this function $y_1(v)$ leads to the MAP estimator:

$$\hat{v}_{map}(y_1; \xi_v) = \text{sign}(y_1) \rho\lambda_1 \max[|y_1| - \kappa\sigma_u, 0] = \rho\lambda_1 [y_1 - \text{sign}(y_1)\kappa\sigma_u] \mathbf{1}_{|y_1| > \kappa\sigma_u}, \quad (66)$$

which has a learning threshold $\kappa\sigma_u = \frac{\rho\lambda\sigma_u^2}{\xi_v}$. Eq. (66) is also known as ‘‘soft-thresholding’’ in statistics. This gives a Bayesian interpretation for the LASSO algorithm. LASSO has a similar objective function that involves an l^1 penalty arising from the Laplace prior. The MAP estimate \hat{v}_{map} is a continuous and piecewise-linear function of y_1 . One can also apply the MAP learning procedure to directly estimate the residual signal $\tilde{\theta} = \tilde{v} - p_1$:

$$\hat{\theta}_{map} = \arg \max_{\theta} \exp \left[-\frac{(y_1 - \frac{\theta + \lambda_1 y_1}{\rho\lambda_1})^2}{2\sigma_u^2} - \frac{|\theta + \lambda_1 y_1|}{\xi_v} \right] = \arg \min_{\theta} \frac{(y_1 - \frac{\theta + \lambda_1 y_1}{\rho\lambda_1})^2}{2\sigma_u^2} + \frac{|\theta + \lambda_1 y_1|}{\xi_v}. \quad (67)$$

The first order condition of this objective leads to

$$y_1(\theta) = \frac{\theta}{\rho\lambda_1} + \text{sign}(\theta) \frac{\rho\kappa\sigma_u}{\rho - 1}. \quad (68)$$

Graphically inverting the function $y_1(\theta)$ yields the MAP estimator of $\tilde{\theta}$:

$$\hat{\theta}_{map} = (\rho - 1)\lambda_1 [y_1 - \text{sign}(y_1)K^*] \mathbf{1}_{|y_1| > K^*}, \quad \text{where } K^* = \frac{\rho\kappa\sigma_u}{\rho - 1} = \frac{\lambda_1\rho^2\sigma_u^2}{(\rho - 1)\xi_v} = \frac{\sqrt{2}\sigma_v}{\lambda_1}. \quad (69)$$

Since $K^* = \frac{\rho}{\rho - 1}\kappa\sigma_u > \kappa\sigma_u$, one can also write $\hat{\theta}_{map} = (\hat{v}_{map} - \lambda_1 y_1) \mathbf{1}_{|y_1| > K^*}$. This establishes an *observational equivalence* to the robust strategy, since we find the following identity

$$Z_{2,n}(s, y_1; K^*) = sZ^\infty(y_1, K^*) \mathbf{1}_{|y_1| > K^*} = s \frac{(\hat{v}_{map} - \lambda_1 y_1) \mathbf{1}_{|y_1| > K^*}}{2(N + 1)\lambda_2} = \frac{s \cdot \hat{\theta}_{map}}{2(N + 1)\lambda_2}. \quad (70)$$

Therefore, if arbitrageurs directly use the MAP rule to estimate the mispricing signal $\tilde{\theta}$, they will get the same strategy $Z_{2,n}(s, y_1; K^*)$. This MAP rule (posterior mode estimate) differs from the posterior mean $\hat{v}(y_1; \xi_v)$ which drives the REE strategy $Z_{2,n}^o(s, y_1; \xi_v)$.

Proof of Corollary 2.4: The MAP estimate for each asset value under the prior $\mathcal{L}(0, \xi_v)$ is:

$$\hat{v}_{i, \text{map}} = \arg \max_{v_i} f(v_i | y_{1,i}) = \exp \left[-\frac{(y_{1,i} - \frac{v_i}{\rho \lambda_1})^2}{2\sigma_u^2} - \frac{|v_i|}{\xi_v} \right] = \arg \min_{v_i} \left| p_{1,i} - \frac{v_i}{\rho} \right|^2 + \frac{2(\lambda_1 \sigma_u)^2}{\xi_v} |v_i|, \quad (71)$$

which amounts to the LASSO objective in the Lagrangian form for $i \in \{1, \dots, M\}$. This leads to the trading algorithm below, which takes the price change $p_{1,i}$ for each stock as input:

$$Z_{2,n}(p_{1,i}, \xi_v) = \frac{(\rho - 1) [\lambda_1 y_{1,i} - \text{sign}(y_{1,i}) \lambda_1 K^*] \mathbf{1}_{|\lambda_1 y_{1,i}| > \lambda_1 K^*}}{2(N+1)\lambda_2} = \frac{\rho - 1}{N+1} \cdot \frac{p_{1,i} \pm 2\xi_v}{2\lambda_2} \mathbf{1}_{|p_{1,i}| > 2\xi_v} \quad (72)$$

where we have used Eq. (16) to derive $\lambda_1 K^* = \sqrt{2}\sigma_v = 2\xi_v$ given $\xi_v = \sigma_v/\sqrt{2}$. Q.E.D.

What if arbitrageurs all adhere to the Gaussian prior? First, they will not trade if their Gaussian prior is identical to market makers' Gaussian prior because they will find out the market is efficient in the semi-strong sense. Arbitrageurs only trade when they have different prior beliefs. Let's model their Gaussian prior as $\tilde{v} \sim \mathcal{N}(0, \tilde{\zeta}^2)$, where $\tilde{\zeta}$ is a random variable reflecting the model uncertainty about the Gaussian prior dispersion. The assumption of prior distribution only changes how arbitrageurs learn from prices without affecting the informed trader's strategy by Assumption 2.1. For any specific value of $\tilde{\zeta} = \zeta$, the arbitrageurs' posterior belief about \tilde{v} conditional on $\tilde{y}_1 = \frac{\tilde{v}}{\rho \lambda_1} + \tilde{u}_1$ is still Gaussian:

$$f(v|y_1) = \frac{f(y_1|v)f_G(v)}{f(y_1)} = \frac{1}{2\pi\zeta\sigma_u f(y_1)} \exp \left[-\frac{(y_1 - v/(\rho\lambda_1))^2}{2\sigma_u^2} - \frac{v^2}{2\zeta^2} \right]. \quad (73)$$

Under the Gaussian prior of \tilde{v} , arbitrageurs believe that $y_1 = \frac{\tilde{v}}{\rho \lambda_1} + \tilde{u}_1 \sim \mathcal{N}(0, \zeta^2/(\rho\lambda_1)^2 + \sigma_u^2)$ for a given value of ζ . By projection theorem, they obtain a linear estimator,

$$\hat{v}(y_1; \zeta) = \mathbb{E}^{\mathcal{N}}[\tilde{v}|y_1, \zeta] = \frac{\zeta^2/(\rho\lambda_1)}{\zeta^2/(\rho\lambda_1)^2 + \sigma_u^2} y_1 = \frac{\rho\lambda_1\zeta^2}{\zeta^2 + (\rho\lambda_1\sigma_u)^2} y_1. \quad (74)$$

The mean of a Gaussian distribution is the same as its mode. So the MAP estimate of \tilde{v} coincides with the posterior mean, i.e., $\hat{v}_{\text{map}} = \hat{v}$ in this case. The rational strategy for arbitrageurs with Gaussian priors is always a linear function of the order flow y_1 :

$$Z_{2,n}^o(y_1; \zeta) = \frac{1}{N+1} \frac{\hat{v} - \lambda_1 y_1}{2\delta\lambda_1} = \frac{(\rho - 1)\zeta^2 - (\rho\lambda_1\sigma_u)^2}{\zeta^2 + (\rho\lambda_1\sigma_u)^2} \cdot \frac{y_1}{2(N+1)}, \quad \text{for } n = 1, \dots, N. \quad (75)$$

Any uncertainty about the prior ζ only changes the slope of this linear strategy. Therefore, the robust strategy must be linear under the max-min choice criteria.

A.5 Proof of Corollary 2.5

If arbitrageurs follow the REE strategy when $s = 1$, the price at $t = 2$ is

$$\tilde{p}_2 = \lambda_1 \tilde{y}_1 + \lambda_2 \left[X_2 + \sum_{n=1}^N Z_{2,n}^o(s, \tilde{y}_1; \xi_v) + \tilde{u}_2 \right] = \frac{\tilde{v} + \lambda_1 \tilde{y}_1}{2} + \frac{N}{N+1} \frac{\hat{v} - \lambda_1 \tilde{y}_1}{2} + \lambda_2 \tilde{u}_2. \quad (76)$$

As $N \rightarrow \infty$, the expectation of $\tilde{p}_2 = P_2(\tilde{y}_1, \tilde{y}_2)$ under arbitrageurs' information and belief is

$$\lim_{N \rightarrow \infty} \mathbb{E}^A[\tilde{p}_2 | \mathcal{I}_{2,z}] = \frac{\hat{v} + \lambda_1 y_1}{2} + \frac{\hat{v} - \lambda_1 y_1}{2} = \hat{v} = \mathbb{E}^A[\tilde{v} | \mathcal{I}_{2,z}]. \quad (77)$$

When arbitrageurs use the robust strategy, the price at $t = 2$ is

$$\tilde{p}_2 = \lambda_1 \tilde{y}_1 + \lambda_2 \left[X_2 + \sum_{n=1}^N Z_{2,n}(s, \tilde{y}_1; K^*) + \tilde{u}_2 \right] = \frac{\tilde{v} + \lambda_1 \tilde{y}_1}{2} + \frac{N}{N+1} \frac{\hat{v}_{map} - \lambda_1 \tilde{y}_1}{2} \mathbf{1}_{|\tilde{y}_1| > K^*} + \lambda_2 \tilde{u}_2. \quad (78)$$

The (ex ante) expected price under arbitrageurs' information and belief has a positive limit:

$$\lim_{N \rightarrow \infty} \mathbb{E}^A[\tilde{p}_2 | \mathcal{I}_{2,z}] = \frac{\hat{v} + \lambda_1 y_1}{2} + \frac{\hat{v}_{map} - \lambda_1 y_1}{2} \mathbf{1}_{|y_1| > K^*} = \frac{\hat{v} + \hat{v}_{map}}{2} - \frac{\hat{v}_{map} - \lambda_1 y_1}{2} \mathbf{1}_{|y_1| > K^*} \neq \hat{v}, \quad (79)$$

indicating price inefficiency in the limit of $N \rightarrow \infty$.

A.6 Proof of Corollary 2.6

If arbitrageurs only trade at $t = 2$ and follow the robust strategy we derived, each of them may find that the total trading of other arbitrageurs has a response slope greater than one, i.e., $\frac{N-1}{N+1} \cdot \frac{1-\mu}{1+\mu} > 1$ if $-1 < \mu < 0$ and $N > -\frac{1}{\mu}$. It may become profitable for any arbitrageur to disrupt the equilibrium by trading a large quantity, $z_1 \gg K^*$, in the first period so that the other arbitrageurs will be triggered almost surely. If $z_1 > \frac{(N-1)(\mu-1)}{2(N\mu+1)} K^*$, the momentum trading of arbitrageurs at $t = 2$ can overwhelm the trade z_1 . This may create opportunities for the initial instigator to unwind her position at favorable prices.

Suppose the n -th arbitrageur (instigator) secretly trades $z_1 \neq 0$ when $s = 1$ to trick other traders. Her objective at $t = 2$ is to maximize the minimum expected profit over all possible priors: $\max_{z'_{2,n} \in \mathcal{Z}} \min_{\xi \in \Omega} \mathbb{E}^A[(\tilde{v} - \lambda_1 \tilde{y}'_1 - \lambda_2 \tilde{y}'_2) z'_{2,n} | \mathcal{I}_{2,z}]$, where $\tilde{y}'_1 = X_1(\tilde{v}) + z_1 + \tilde{u}_1$ and $\tilde{y}'_2 = X_2(\tilde{v}, \tilde{y}'_1) + z'_{2,n} + Z_{2,-n}(\tilde{y}'_1, K^*) + \tilde{u}_2$. Here, $Z_{2,-n} = \sum_{m \neq n} Z_{2,m}(y'_1, K^*) = \frac{(N-1)\hat{\theta}_{map}(y'_1)}{2(N+1)\lambda_2}$ is the total quantity traded by the other arbitrageurs (excluding the n -th one) who form the estimate of $\tilde{\theta} = \tilde{v} - \lambda_1 y'_1$ based on y'_1 without knowing that y'_1 contains the secret trade z_1 . The instigator's estimate, $\hat{\theta}_{map}(y_1) = [\hat{v}_{map}(y_1) - \lambda_1 y_1] \mathbf{1}_{|y_1| > K^*} = (\rho - 1) \lambda_1 [y_1 - \text{sign}(y_1) K^*] \mathbf{1}_{|y_1| > K^*}$,

is however based on $y_1 = x_1 + u_1$ instead of y'_1 , because she is aware of the order flow z_1 secretly placed by herself. The strategy of this instigator in the second period reflects how she strategically exploits the other traders' overreaction due to her trade z_1 :

$$\begin{aligned}
Z'_{2,n}(y_1, z_1) &= \frac{\hat{v}_{map}(y_1) - \lambda_1 y'_1}{4\lambda_2} \mathbf{1}_{|y_1| > K^*} - \frac{N-1}{4(N+1)\lambda_2} [\hat{v}_{map}(y'_1) - \lambda_1 y'_1] \mathbf{1}_{|y'_1| > K^*} \\
&= \frac{\hat{\theta}_{map}(y_1)}{4\lambda_2} - \frac{z_1}{4\delta} - \frac{(N-1)(\rho-1)(y_1 + z_1 - K^*)}{4(N+1)\delta} \\
&= \frac{\hat{\theta}_{map}(y_1)}{2(N+1)\lambda_2} - \frac{(N+1)z_1 + (N-1)(\rho-1)[z_1 + (y_1 - K^*)\mathbf{1}_{|y_1| < K^*}]}{4(N+1)\delta}, \tag{80}
\end{aligned}$$

where we used the condition $z_1 \gg K^*$ so that $\mathbf{1}_{|y'_1=y_1+z_1| > K^*} = 1$ with probability arbitrarily close to 1. Her expected total profit is $\Pi_{z,n}^d = \mathbb{E}^A[(\tilde{v} - \lambda_1 \tilde{y}'_1)z_1 + (\tilde{v} - \lambda_1 \tilde{y}'_1 - \lambda_2 \tilde{y}'_2) \cdot Z'_{2,n} | \mathcal{I}_{1,z}]$ and the extra profit attributable to her unilateral deviation $(z_1, Z'_{2,n})$ is

$$\Delta \Pi_{z,n}^d = \Pi_{z,n}^d - \mathbb{E}^A[(\tilde{v} - \lambda_1 \tilde{y}_1 - \lambda_2 \tilde{y}_2) \cdot Z_{2,n} | \tilde{s} = 1], \tag{81}$$

where $\tilde{y}_1 = X_1(\tilde{v}) + \tilde{u}_1$, $\tilde{y}_2 = X_2(\tilde{v}, \tilde{y}_1) + \sum_{n=1}^N Z_{2,n}(\tilde{y}_1, K^*) + \tilde{u}_2$, and $Z_{2,n}(\tilde{y}_1, K^*) = \frac{\hat{\theta}_{map}(\tilde{y}_1)}{2(N+1)\lambda_2}$. Using the results $\mathbb{E}^A[\tilde{y}_1 \cdot z_1] = 0$, $\mathbb{E}^A[\hat{\theta}_{map}(\tilde{y}_1) \cdot z_1] = 0$ and $\hat{\theta}_{map} \mathbf{1}_{|y_1| < K^*} = 0$, we derive that

$$\begin{aligned}
\Delta \Pi_{z,n}^d &= -\lambda_1 z_1^2 + \lambda_2 \mathbb{E}^A[(Z'_{2,n}(\tilde{y}_1, z_1))^2] - \lambda_2 \mathbb{E}^A[(Z_{2,n}(\tilde{y}_1, K^*))^2] \\
&= -\lambda_1 z_1^2 + \lambda_2 \mathbb{E}^A[(Z'_{2,n}(\tilde{y}_1, z_1) + Z_{2,n}(\tilde{y}_1, K^*))(Z'_{2,n}(\tilde{y}_1, z_1) - Z_{2,n}(\tilde{y}_1, K^*))] \\
&= -\lambda_1 z_1^2 + \lambda_2 \left(\frac{\lambda_1}{\lambda_2} \right)^2 \mathbb{E}^A \left[\left(\frac{(N-1)\rho + 2}{4(N+1)} z_1 + \frac{(N-1)(\rho-1)}{4(N+1)} (\tilde{y}_1 - K^*) \mathbf{1}_{|\tilde{y}_1| < K^*} \right)^2 \right] \\
&= -\lambda_1 z_1^2 + (1-\mu)\lambda_1 z_1^2 \left[\frac{(N-1)\rho + 2}{4(N+1)} \right]^2 + (1-\mu)\lambda_1 \frac{(N-1)^2(\rho-1)^2}{16(N+1)^2} \mathbb{E}^A[(\tilde{y}_1 - K^*)^2 \mathbf{1}_{|\tilde{y}_1| < K^*}].
\end{aligned}$$

Since $\delta = \frac{1}{1-\mu}$ and $\rho = \frac{3+\mu}{1+\mu}$ by definition, the above expression is positive if the coefficient in front of z_1^2 is positive. This is equivalent to the condition:

$$1 + \frac{N-1}{N+1} \cdot \frac{2}{1+\mu} > \frac{4}{\sqrt{1-\mu}}. \tag{82}$$

Given any $N > 1$, there exists a critical liquidity point $\mu^*(N)$ below which $\Delta \Pi_{z,n}^d > 0$. For example, $\mu^*(N=2) \approx -0.68037$, $\mu^*(N=3) \approx -0.54843$, $\mu^*(N=10) \approx -0.33525$, $\lim_{N \rightarrow \infty} \mu^* = \mu_\epsilon \approx -0.23191$. Thus, in the liquidity regime $\mu < \mu_\epsilon \approx -0.23191$, if the number of arbitrageurs is large enough, the conjectured equilibrium $\mathbf{Z} = [\langle 0, Z_{2,1} \rangle, \dots, \langle 0, Z_{2,N} \rangle]$ may fail, because it may permit profitable deviations (or disruptive strategies) at $t = 1$.

A.7 Savvy Informed Trader: Rational-Expectations Equilibrium

For $s = 1$, we investigate the *rational-expectations equilibrium* (REE) in the model of savvy informed trader who anticipates arbitrageurs and strategically interacts with them. Based on $\mathcal{I}_{2,x} = \{v, s, y_1\}$, the informed trader conjectures her residual demand at $t = 2$ and solves

$$X_2(v, y_1) = \arg \max_{x_2} \mathbb{E}[(v - P_2(\tilde{y}_1, \tilde{y}_2)) x_2 | \mathcal{I}_{2,x}] = (1 - \mu) \frac{v - \lambda_1 y_1}{2\lambda_1} - \frac{\mathbb{E}[Z_2 | \mathcal{I}_{2,x}]}{2}. \quad (83)$$

As the informed trader takes into account the price impact of all arbitrageurs, she will reduce her trading quantity by one half of the total arbitrage trading that she expects at $t = 2$. The information set of arbitrageurs right after $t = 1$ is $\mathcal{I}_{2,z} = \{s, y_1\}$, which is nested into the informed trader's information set $\mathcal{I}_{2,x} = \{v, s, y_1\}$. The n -th arbitrageur's objective is

$$\max_{z_{2,n}} \mathbb{E}[z_{2,n} (\tilde{v} - \lambda_1 \tilde{y}_1 - \lambda_2 [X_2(\tilde{v}, \tilde{y}_1) + z_{2,n} + Z_{2,-n}(\tilde{y}_1) + \tilde{u}_2]) | \mathcal{I}_{2,z}], \quad (84)$$

from which she can solve the optimal strategy as below

$$Z_{2,n}(y_1) = (1 - \mu) \frac{\hat{v} - \lambda_1 y_1}{4\lambda_1} - \frac{\mathbb{E}[Z_{2,-n} | \mathcal{I}_{2,z}]}{2} + \frac{\mathbb{E}[\mathbb{E}[Z_2 | \mathcal{I}_{2,x}] | \mathcal{I}_{2,z}]}{4}. \quad (85)$$

Arbitrageurs are symmetric in terms of their information and objectives. The n -th arbitrageur conjectures that the other arbitrageurs will trade $Z_{2,m} = \eta \cdot (\hat{v} - \lambda_1 y_1)$ for $m = 1, \dots, N$ and $m \neq n$, and she also conjectures the informed trader's conjecture that all arbitrageurs trade symmetrically $Z_{2,n} = \eta \cdot (\hat{v} - \lambda_1 y_1)$ for $n = 1, \dots, N$. So her optimal strategy becomes

$$Z_{2,n}(y_1) = \left(\frac{1 - \mu}{4\lambda_1} - \frac{(N - 1)\eta}{2} + \frac{N\eta}{4} \right) (\hat{v} - \lambda_1 y_1). \quad (86)$$

In a symmetric equilibrium, every arbitrageur conjectures in the same way and solves the same problem. This symmetry requires $\eta = \frac{1 - \mu}{4\lambda_1} - \frac{(N - 1)\eta}{2} + \frac{N\eta}{4}$ that has a unique solution $\eta = \frac{1 - \mu}{(N + 2)\lambda_1}$. Thus the total order flow from arbitrageurs at $t = 2$ can be written as

$$Z_2 = \sum_{n=1}^N Z_{2,n} = N\eta \cdot (\hat{v} - \lambda_1 y_1) = \frac{N(\hat{v} - \lambda_1 y_1)}{(N + 2)\lambda_2}. \quad (87)$$

One can prove a simple result that $Z_{2,n} = \mathbb{E}[X_2 | \mathcal{I}_{2,z}]$, i.e., every arbitrageur expects that the informed trader on average trades the same quantity as she does. By Eq. (83) and (87),

$$\mathbb{E}[X_2(\tilde{v}, \tilde{y}_1) | \mathcal{I}_{2,z}] = \frac{\mathbb{E}[\tilde{v} | \mathcal{I}_{2,z}] - \lambda_1 y_1}{2\lambda_2} - \frac{\mathbb{E}[\mathbb{E}[Z_2 | \mathcal{I}_{2,x}] | \mathcal{I}_{2,z}]}{2} = \frac{\hat{v} - \lambda_1 y_1}{2\lambda_2} - \frac{Z_2}{2} = \frac{Z_2}{N} = Z_{2,n}. \quad (88)$$

As $\hat{v} = E[\tilde{v}|\mathcal{I}_{2,z}]$, we obtain the following

$$Z_{2,n}(y_1) = \eta(\hat{v} - \lambda_1 y_1) = \frac{1 - \mu}{(N + 2)\lambda_1} (E[\tilde{v}|\mathcal{I}_{2,z}] - \lambda_1 y_1) = E[X_2|\mathcal{I}_{2,z}], \quad (89)$$

$$X_2(v, y_1) = \frac{v - \lambda_1 y_1}{2\lambda_2} - \frac{Z_2}{2} = \frac{v - \lambda_1 y_1}{2\lambda_2} - \frac{N}{N + 2} \frac{E[\tilde{v}|\mathcal{I}_{2,z}] - \lambda_1 y_1}{2\lambda_2}. \quad (90)$$

One can rewrite the second-period informed trading strategy as

$$X_2(v, y_1) = \frac{v - \lambda_1 y_1}{(N + 2)\lambda_2} + \frac{N}{N + 2} \cdot \frac{v - \hat{v}}{2\lambda_2}, \quad (91)$$

where the first term is proportional to her informational advantage over market makers and the second term is proportional to her residual advantage over arbitrageurs. Let $\hat{v} = E[\tilde{v}|\mathcal{I}_{2,z}] = g(y_1)$. The informed trader will conjecture the average price at $t = 2$ to be

$$E[\tilde{p}_2|\mathcal{I}_{2,x}] = E \left[\lambda_1 \tilde{y}_1 + \lambda_2 \left(X_2 + \sum_{n=1}^N Z_{2,n} + \tilde{u}_2 \right) \middle| \mathcal{I}_{2,x} \right] = \frac{(N + 2)v + Ng(y_1) + 2\lambda_1 y_1}{2(N + 2)}, \quad (92)$$

The informed trader's expected profit from her second-period trading is

$$\Pi_{2,x}(v, y_1) = E[x_2(v - \tilde{p}_2)|\mathcal{I}_{2,x}] = \frac{1}{\lambda_2} \left(\frac{(N + 2)v - Ng(y_1) - 2\lambda_1 y_1}{2(N + 2)} \right)^2. \quad (93)$$

The informed trader needs to choose x_1 that maximizes her total expected profits:

$$\begin{aligned} \Pi_x(v) &= \max_{x_1} E[x_1(v - \lambda_1 \tilde{y}_1) + \Pi_{2,x}(v, \tilde{y}_1)|\mathcal{I}_{1,x}] \\ &= \max_{x_1} x_1(v - \lambda_1 x_1) + \frac{1 - \mu}{\lambda_1} E \left[\left(\frac{(N + 2)v - Ng(\tilde{y}_1) - 2\lambda_1 \tilde{y}_1}{2(N + 2)} \right)^2 \middle| \mathcal{I}_{1,x} \right], \end{aligned} \quad (94)$$

where $\mathcal{I}_{1,x} = \{v, s = 1\}$. As regularity conditions permit, one can interchange expectation and differentiation operations to derive the first order condition (FOC) for $x_1 = X_1(v)$:

$$0 = v - 2\lambda_1 x_1 - \frac{1 - \mu}{\lambda_1} E \left[\frac{(N + 2)v - Ng(x_1 + \tilde{u}_1) - 2\lambda_1(x_1 + \tilde{u}_1)}{2(N + 2)} \cdot \frac{Ng'(x_1 + \tilde{u}_1) + 2\lambda_1}{N + 2} \right]. \quad (95)$$

When $s = 1$, there does not exist a linear REE where the informed trader's strategy X_1 is a linear function of v . This is proved by contradiction: Suppose X_1 is a linear function of v , the posterior mean $g(y_1) = E[\tilde{v}|\mathcal{I}_{2,z}]$ will be a nonlinear function of y_1 . With a nonlinear $g(y_1)$, the FOC Eq. (95) does not permit a linear solution to $X_1(v)$. Nonlinearity makes Eq. (95) and the REE intractable in general.

A.8 Savvy Informed Trader: Asymptotic Linearity

Based on the asymptotic conjecture of $X_1(v) \rightarrow \frac{v}{\rho\lambda_1} + c\kappa\sigma_u$ in the high signal regime, arbitrageurs will find that the posterior distribution of x_1 conditional on y_1 is asymptotically

$$f(x_1|y_1) \rightarrow \frac{\rho\lambda_1}{\xi_v f(y_1) \sqrt{2\pi\sigma_u^2}} \exp \left[-\frac{(y_1 - x_1)^2}{2\sigma_u^2} - \frac{\rho\lambda_1(x_1 - c\kappa\sigma_u)}{\xi_v} \right]. \quad (96)$$

At large order flows, it is deduced that $E[\tilde{x}_1|y_1] \rightarrow y_1 - \kappa\sigma_u$ and furthermore

$$E[\tilde{v}|\mathcal{I}_{2,z}] \rightarrow \rho\lambda_1[y_1 - (1+c)\kappa\sigma_u]. \quad (97)$$

This result makes the informed trader's FOC Eq. (95) for $x_1 = X_1(v)$ linear again:

$$0 = v - 2\lambda_1 x_1 - \frac{1}{\lambda_2} E \left[\left(\frac{(N+2)v - N\rho\lambda_1[\tilde{y}_1 - (1+c)\kappa\sigma_u] - 2\lambda_1\tilde{y}_1}{2(N+2)} \right) \left(\frac{N\rho\lambda_1 + 2\lambda_1}{N+2} \right) \middle| \mathcal{I}_{1,x} \right]. \quad (98)$$

After some calculation with the notation $\delta \equiv \frac{\lambda_2}{\lambda_1} = \frac{1}{1-\mu}$, we get

$$0 = v - 2\lambda_1 x_1 - \frac{N\rho + 2}{2\delta(N+2)^2} [(N+2)v - (N\rho + 2)\lambda_1 x_1 + N(1+c)\kappa\rho\lambda_1\sigma_u]. \quad (99)$$

This FOC leads to a linear expression of x_1 which conforms to the original linear conjecture:

$$X_1(v) = \frac{(N+2)[2\delta(N+2) - N\rho - 2]}{4\delta(N+2)^2 - (N\rho + 2)^2} \left(\frac{v}{\lambda_1} \right) - \frac{N\rho(N\rho + 2)(1+c)\kappa}{4\delta(N+2)^2 - (N\rho + 2)^2} \sigma_u. \quad (100)$$

Matching the first term leads to a quadratic equation for ρ :

$$-2(\rho - 1)(N\rho + 2) + 2\delta(\rho - 2)(N+2)^2 = 0. \quad (101)$$

There are two roots to this equation but only one of them is sensible as it increases with δ :

$$\rho(\delta, N) = \frac{N + \delta(N+2)^2 - 2 - (N+2)\sqrt{\delta^2(N+2)^2 - 2\delta(3N+2) + 1}}{2N}. \quad (102)$$

Substituting $\delta = \frac{1}{1-\mu}$ into the above equation leads to

$$\rho(\mu, N) = \frac{2 + 5N + N^2 + 2\mu - N\mu - (N+2)\sqrt{N^2 + (1+\mu)^2 + 2N(3\mu - 1)}}{2N(1-\mu)}. \quad (103)$$

For $N = 0$, we have $\rho = \frac{3+\mu}{1+\mu}$ which is identical to the parameter ρ in the previous model. There are two more useful limits: $\lim_{\mu \rightarrow 0} \rho = 2 \left(1 + \frac{1}{N}\right)$ and $\lim_{\mu \rightarrow 1} \rho = 2$. This equilibrium parameter ρ decreases with μ and N . It is bounded in the range $\left[2, \frac{2(N+1)}{N}\right]$. Now we match the intercept terms and utilize the slope-matching relation to obtain

$$c = -\frac{N(2 + N\rho)}{2\delta(2 + N)^2 - 2(2 + N\rho)} = -\frac{3 + N - \mu - \sqrt{N^2 + (1 + \mu)^2 + 2N(3\mu - 1)}}{1 + N + \mu + \sqrt{N^2 + (1 + \mu)^2 + 2N(3\mu - 1)}} \cdot \frac{N}{2}. \quad (104)$$

In the competitive case, we have

$$\lim_{N \rightarrow \infty} c = \lim_{N \rightarrow \infty} \frac{-N(2 + 2N)}{2\delta(2 + N)^2 - 2(2 + 2N)} = -\frac{1}{\delta} = -(1 - \mu). \quad (105)$$

There are two more useful limits: $\lim_{\mu \rightarrow 0} c = -1$ and $\lim_{\mu \rightarrow 1} c = 0$.

Approximation to the rational equilibrium. The symmetry indicates that $X_1(-v) = -X_1(v)$. If $X_1(v)$ is monotone, it should cross the origin and be roughly linear in that neighborhood. With the linearized conjecture $X_1(v \rightarrow 0) \rightarrow \frac{v}{\alpha\lambda_1}$, one can use Taylor expansion of Eq. (14) at small y_1 to approximate $E[\tilde{v}|y_1 \ll \kappa\sigma_u] \approx \alpha\beta\lambda_1 y_1$, where α and β are determined by

$$\beta N[\beta N - (N + 2)]\alpha^2 + 2 \left(\frac{(N + 2)^2}{1 - \mu} + 2\beta N - (N + 2) \right) \alpha - 4 \left(\frac{(N + 2)^2}{1 - \mu} - 1 \right) = 0,$$

$$\beta = 1 + \left(\frac{\alpha\lambda_1\sigma_u}{\xi} \right)^2 - \left(\frac{\alpha\lambda_1\sigma_u}{\xi} \right) \frac{e^{-\frac{(\alpha\lambda_1\sigma_u)^2}{2\xi^2}} \sqrt{\frac{2}{\pi}}}{\operatorname{erfc}\left(\frac{\alpha\lambda_1\sigma_u}{\sqrt{2}\xi}\right)}.$$

The first equation is derived from the FOC Eq. (95) and the second one is from the Taylor expansion of Eq. (14). Given $\{\mu, N, \xi\}$, one can numerically find a unique pair of positive solutions to α and β . With constant depth ($\mu = 0$), the first equation becomes $\alpha = \frac{2(N+3)}{N+2-N\beta}$ and the total demand from arbitrageurs becomes $\lim_{\mu \rightarrow 0} Z_2 \approx \lim_{\mu \rightarrow 0} \frac{N(\alpha\beta-1)}{N+2} y_1 = (\alpha - 3)y_1$ for small y_1 . The rational equilibrium is not tractable, but one can approximate the arbitrageurs' rational strategy by smoothly pasting the two regimes of asymptotic linearity. There are different methods to make a smooth transition between two linear segments; for example, any sigmoid functions that approach the Heaviside function may work. Here, I use $q(y) = \frac{1}{2}\operatorname{erfc}[a(\kappa\sigma_u - y)] + \frac{1}{2}\operatorname{erfc}[a(\kappa\sigma_u + y)]$, with a tunable parameter $a > 0$ and approximate the posterior mean estimate of \tilde{v} by

$$\hat{v}_a(y_1) \approx [1 - q(y_1)]\alpha\beta\lambda_1 y_1 + q(y_1)\rho\lambda_1[y_1 - \operatorname{sign}(y_1)(1 + c)\kappa\sigma_u]. \quad (106)$$

Clearly, $\hat{v}_a \rightarrow \alpha\beta\lambda_1 y_1$ at $|y_1| \ll \kappa\sigma_u$ and $\hat{v}_a \rightarrow \rho\lambda_1[y_1 - \text{sign}(y_1)(1+c)\kappa\sigma_u]$ at $|y_1| \gg \kappa\sigma_u$. The figure below shows numerical approximations to the Bayesian-rational strategy $Z_{2,n}^o(s=1, y_1; \xi)$ under different ξ , compared with the linear-triggering strategy $Z_{2,n}(s=1, y_1; K^*)$.



Figure 10. Approximate rational strategies and the linear-triggering strategy (red line).

A.9 Learning Bias and Strategic Informed Trading

Corollary A.1. *Arbitrageurs tend to underestimate the private signal \tilde{v} by a negative amount $-\rho\kappa\lambda_1\sigma_u < 0$. Anticipating this estimation bias, the informed trader in the high signal regime will strategically shift her demand downward by an amount of $c\kappa\sigma_u < 0$ at $t=1$ and upward by an amount of $d\kappa\sigma_u > 0$ at $t=2$ where the parameter $d(\mu, N)$ is given by Eq. (111). her average terminal position contains an informational component and a strategic component, that is, $E[X_1(v) + X_2(v, \tilde{u}_1)] \rightarrow X_{inf}^*(v) + X_{str}^*$, where $X_{str}^* = (c+d)\kappa\sigma_u$ and*

$$X_{inf}^* = \frac{N+1+\mu+\rho(1-\mu)}{N+2} \frac{v}{\rho\lambda_1}, \quad (107)$$

Given any $N > 0$, the maximum of $X_{inf}^*(v)$ is at $\mu_c(N) = \sqrt{N(N+2)^3} - N(N+3) - 1$.

Proof: In the asymptotic rational equilibrium we have shown $E[\tilde{v}|\mathcal{I}_{2,z}] \rightarrow \rho\lambda_1[y_1 - (1+c)\kappa\sigma_u]$ and $y_1 = X_1(v) + \tilde{u}_1 \rightarrow (\rho\lambda_1)^{-1}\tilde{v} + c\kappa\sigma_u + \tilde{u}_1$. Arbitrageurs tend to underestimate \tilde{v} ,

$$E[\tilde{v}|\mathcal{I}_{2,z}] - \tilde{v} = -\rho\lambda_1\kappa\sigma_u + \rho\lambda_1\tilde{u}_1 \sim \mathcal{N}[-\rho\lambda_1\kappa\sigma_u, (\rho\lambda_1\sigma_u)^2], \quad (108)$$

which has a negative mean $-\rho\lambda_1\kappa\sigma_u < 0$. This learning bias of arbitrageurs entices the informed trader to strategically exploit it. This can be seen from her asymptotic strategy:

$$X_2(v, y_1) \rightarrow (1-\mu) \left[\frac{v - \lambda_1 y_1}{2\lambda_1} - \frac{N}{N+2} \frac{(\rho-1)y_1 - (1+c)\kappa\rho\sigma_u}{2} \right], \quad (109)$$

whose average contains both an informational component and a strategic one:

$$E[X_2|\tilde{v} = v] = \frac{(1-\mu)(1-\rho^{-1})}{\lambda_1(N+2)}v + \frac{(1-\mu)(N\rho-2c)}{2(N+2)}\kappa\sigma_u. \quad (110)$$

We define another parameter d for this strategic shift which decreases with μ and N :

$$d(\mu, N) = \frac{(1-\mu)(N\rho-2c)}{2(N+2)} = \frac{2N(1-\mu)}{1+N+\mu+\sqrt{N^2+(1+\mu)^2+2N(3\mu-1)}}. \quad (111)$$

It has the following limit results: $\lim_{\mu \rightarrow 0} d = 1$, $\lim_{\mu \rightarrow 1} d = 0$, and $\lim_{N \rightarrow \infty} d = 1 - \mu$. Thus, we have shown that $X_1 \rightarrow \frac{v}{\rho\lambda_1} + c\kappa\sigma_u$ where $c < 0$ and $E[X_2|v] \rightarrow \frac{(1-\mu)(1-\rho^{-1})}{\lambda_1(N+2)}v + d\kappa\sigma_u$ where $d > 0$. This shows how the informed trader strategically exploit the arbitrageurs' bias $\kappa\sigma_u$.

The asymptotic terminal position of the informed trader can be decomposed into an informational term and a strategic term, that is, $E[X_1(v) + X_2(v, \tilde{u}_1)] \rightarrow X_{inf}^*(v) + X_{str}^*$ where $X_{str}^* = (c+d)\kappa\sigma_u \geq 0$. The information-based target inventory is found to be

$$\begin{aligned} X_{inf}^*(v; \mu, N) &= \frac{v}{\rho\lambda_1} + \frac{(1-\mu)(1-\rho^{-1})}{\lambda_1(N+2)}v = \frac{N+1+\mu+\rho(1-\mu)}{N+2} \cdot \frac{v}{\rho\lambda_1} \\ &= \frac{1+3N+\mu-\sqrt{N^2+(1+\mu)^2+2N(3\mu-1)}}{N\rho} \cdot \frac{v}{2\lambda_1}. \end{aligned} \quad (112)$$

which is hump-shaped and reaches its maximum at

$$\mu_c(N) = \sqrt{N(N+2)^3} - N(N+3) - 1. \quad (113)$$

For example, X_{inf}^* has its maximum $0.5359\frac{v}{\lambda_1}$ at $N=1$ and $\mu_c(N=1) = 3\sqrt{3}-5 = 0.196152$. The informed trader manages to reach an informational target position roughly equal to $\frac{v}{2\lambda_1}$.



Figure 11. The information-based target inventory $X_{inf}^*(v)$ and the strategic position X_{str}^* .

A.10 Proof of Proposition 3.1

The candidate linear-triggering strategy for each arbitrageur along the REE asymptotes is

$$Z_{2,n}(s, y_1; K_n) = sZ^\infty(y_1, K^*)\mathbf{1}_{|y_1|>K_n} = s\frac{(1-\mu)(\rho-1)}{N+2} \left[y_1 - \text{sign}(y_1)\frac{\rho(1+c)\kappa\sigma_u}{\rho-1} \right] \mathbf{1}_{|y_1|>K_n}. \quad (114)$$

For $s = 1$, this can be rewritten as

$$\begin{aligned} Z_{2,n}(s, y_1; K_n) &= \frac{\rho\lambda_1[y_1 - \text{sign}(y_1)(1+c)\kappa\sigma_u] - \lambda_1y_1}{(N+2)\lambda_2} \mathbf{1}_{|y_1|>K_n} \\ &= \eta \cdot [\hat{v}_T(y_1; \xi_v) - \lambda_1y_1] \mathbf{1}_{|y_1|>K_n}, \end{aligned} \quad (115)$$

where $\eta = \frac{1-\mu}{(N+2)\lambda_1}$ and the implied learning rule for \tilde{v} is

$$\hat{v}_T(y_1; \xi_v) = \rho\lambda [y_1 - \text{sign}(y_1)(1+c)\kappa\sigma_u] \mathbf{1}_{|y_1|>\kappa\sigma_u}. \quad (116)$$

The learning threshold $\kappa\sigma_u$ here ensures that \hat{v}_T takes the same sign as y_1 .

Now I prove that in equilibrium every arbitrageur will choose the same threshold

$$K^* = \max \left[\kappa\sigma_u, \frac{\rho(1+c)\kappa\sigma_u}{\rho-1} \right]. \quad (117)$$

Intuitively, any trader choosing K_n lower than the learning threshold $\kappa\sigma_u$ may take actions to trade over the states $|y_1| \in [K_n, \kappa\sigma_u]$ where she actually learns nothing under her learning rule, i.e., $\hat{v}_T = 0$ for $|y_1| \in [K_n, \kappa\sigma_u]$. To exclude irrational trading when the inferred signal is zero, the equilibrium threshold must have a lower bound $\kappa\sigma_u$. On the other hand, any trader choosing K_n lower than the intercept $\frac{\rho(1+c)\kappa\sigma_u}{\rho-1}$ may trade against the price trend (contrarian trading) over the states $|y_1| \in [K_n, \frac{\rho(1+c)\kappa\sigma_u}{\rho-1}]$. This may go against the true (fat-tail) signal and incur losses on average. Therefore, the condition $K_n \geq \max \left[\kappa\sigma_u, \frac{\rho(1+c)\kappa\sigma_u}{\rho-1} \right]$ could make arbitrageurs dedicate to the momentum trading strategy which is desirable in our fat-tail setup. When traders choose thresholds, they actually engage in Bertrand-type competition: each of them will keep undercutting the threshold as long as it is more profitable than the case she follows the common threshold used by other traders. Under this competition, the equilibrium threshold is the boundary K^* given by Eq. (117).

Let's first show that to use any threshold K' lower than K^* cannot be an equilibrium. It suffices to show that when everyone else uses $K_{-n} = K' < K^*$, it is a profitable deviation for

the n -th trader to choose $K_n = K^*$. We need to compare the difference of expected profits:

$$\begin{aligned}
& \mathbb{E} [\tilde{\pi}_{z,n}(\tilde{y}_1; K_n = K^*, K_{-n} = K') - \tilde{\pi}_{z,n}(\tilde{y}_1; K_n = K', K_{-n} = K') | \tilde{y}_1 = y_1] \\
= & \mathbb{E} \left[\left\{ -\eta \left(\frac{1}{2} - \frac{\lambda_1 \eta (N-2)}{2(1-\mu)} \right) (\hat{v}_T - \lambda_1 y_1)^2 - \eta (\hat{v}_T - \lambda_1 y_1) \frac{\tilde{v} - \hat{v}_T}{2} \right\} \mathbf{1}_{K' < |y_1| < K^*} \middle| y_1 \right] \\
= & -\frac{\eta}{2} \left[\frac{4\Theta^2}{N+2} + (\hat{v} - \hat{v}_T)\Theta \right] \mathbf{1}_{K' < |y_1| < K^*}, \tag{118}
\end{aligned}$$

where $\Theta = \hat{v}_T - \lambda_1 y_1$ is negative for $K' < y_1 < K^*$ and positive for $K' < -y_1 < K^*$. For the case $K^* = \kappa\sigma_u$, we have $\hat{v}_T = 0$ but $\hat{v} \geq 0$ for $|y_1| \in [K', \kappa\sigma_u]$. It means the last expression of Θ is a parabola that opens downward and crosses the origin. Since Θ takes the opposite sign of y_1 and \hat{v} for $K' < |y_1| < K^*$, the last expression is strictly positive for $K' < |y_1| < K^*$. Similar arguments can be applied to the case $K^* = \frac{\rho(1+c)\kappa\sigma_u}{\rho-1}$. Therefore, $\mathbb{E} [\tilde{\pi}_{z,n}(\tilde{y}_1; K_n = K^*, K_{-n} = K') - \tilde{\pi}_{z,n}(\tilde{y}_1; K_n = K', K_{-n} = K')] > 0$ for $K' < K^*$, i.e., any threshold less than K^* cannot be an equilibrium threshold.

Similarly, any threshold K' larger than K^* cannot be an equilibrium threshold either. As before, it suffices to show that the deviation is profitable for any trader by just choosing $K_n = K^*$ less than K' used by others. The payoff difference given y_1 is positive as well:

$$\begin{aligned}
& \mathbb{E} [\tilde{\pi}_{z,n}(\tilde{y}_1; K_n = K^*, K_{-n} = K') - \tilde{\pi}_{z,n}(\tilde{y}_1; K_n = K', K_{-n} = K') | \tilde{y}_1 = y_1] \\
= & \mathbb{E} \left[\eta (\hat{v}_T - \lambda_1 y_1) \mathbf{1}_{K^* < |y_1| < K'} \left[\frac{\tilde{v} - \lambda_1 y_1}{2} - \frac{\lambda_1 \eta (\hat{v}_T - \lambda_1 y_1)}{1-\mu} \right] \middle| y_1 \right] \\
= & \frac{\eta}{2} \left[\frac{N}{N+2} \Theta^2 + (\hat{v} - \hat{v}_T)\Theta \right] \mathbf{1}_{K^* < |y_1| < K'} > 0. \tag{119}
\end{aligned}$$

It rules out any threshold larger than K^* to be an equilibrium. So the only possible equilibrium choice is K^* . When every trader uses the same threshold K^* , no one will deviate.

Now look at the informed trader in this algorithmic trading game. If arbitrageurs all use the same threshold K (which can be general), the informed trader at $t = 2$ will trade

$$X_2(v, y_1; K) = (1-\mu) \frac{v - \lambda_1 y_1}{2\lambda_1} - s \frac{N(1-\mu)(\rho-1)}{2(N+2)} \left[y_1 - \text{sign}(y_1) \frac{\rho(1+c)\kappa\sigma_u}{\rho-1} \right] \mathbf{1}_{|y_1| > K}, \tag{120}$$

and pick $X_1(v, K)$ that maximizes her total payoff. The price at $t = 2$ can be written as

$$\tilde{p}_2 = \lambda_1 y_1 + \lambda_2 (x_2 + Z_2 \mathbf{1}_{|y_1| > K} + \tilde{u}_2) = \begin{cases} \frac{(N+2)\tilde{v} + 2\lambda_1 y_1 + N\hat{v}_T}{2(N+2)} + \lambda_2 \tilde{u}_2, & \text{if } |y_1| > K \\ \frac{\tilde{v} + \lambda_1 y_1}{2} + \lambda_2 \tilde{u}_2, & \text{if } |y_1| < K, \end{cases} \tag{121}$$

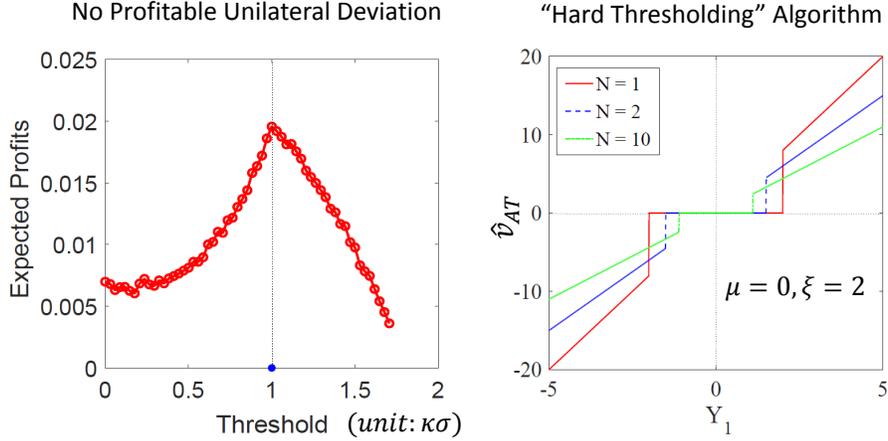


Figure 12. Left: Trading profits if someone deviates from K^* . Right: Learning rule $\hat{v}_T(y_1)$.

depending on whether the arbitrageurs are triggered. The informed trader's expected profit in the second period is also contingent on the state of arbitrageurs:

$$\Pi_{2,x}(v, y_1; K) = \mathbb{E}[x_2(v - \tilde{p}_2) | \mathcal{I}_{2,x}] = \begin{cases} \frac{1}{\lambda_2} \left[\frac{(N+2)v - N\hat{v}_T(y_1) - 2\lambda_1 y_1}{2(N+2)} \right]^2, & \text{if } |y_1| > K \\ \frac{1}{\lambda_2} \left(\frac{v - \lambda_1 y_1}{2} \right)^2, & \text{if } |y_1| < K. \end{cases} \quad (122)$$

Note that her expected profit in the second period is always positive because the informed trader fully anticipates the response of arbitrageurs. The informed trader needs to determine $x_1 = X_1(v; K)$ that maximizes the total expected profit from both periods. The calculation of her total profit, conditional on the private signal v , can be decomposed into three components:

$$\begin{aligned} \Pi_x(v, x_1; K) &= \max_{x_1} \mathbb{E} \left[\Pi_{1,x} + \Pi_{2,x} \mathbf{1}_{|\tilde{y}_1| < K} + \Pi_{2,x} \mathbf{1}_{|\tilde{y}_1| > K} | \mathcal{I}_{1,x} \right] \\ &= x_1(v - \lambda_1 x_1) + \mathbb{E} \left[\frac{(v - \lambda_1(x_1 + \tilde{u}_1))^2}{4\lambda_2} \mathbf{1}_{|x_1 + \tilde{u}_1| < K} \middle| \mathcal{I}_{1,x} \right] \\ &\quad + \mathbb{E} \left[\frac{[(N+2)v - N\hat{v}_T(x_1 + \tilde{u}_1) - 2\lambda_1(x_1 + \tilde{u}_1)]^2}{4(N+2)^2\lambda_2} \mathbf{1}_{|x_1 + \tilde{u}_1| > K} \middle| \mathcal{I}_{1,x} \right] \end{aligned} \quad (123)$$

On one hand, the informed trader may want to trade less to avoid triggering arbitrageurs and take full advantage of her information at $t = 2$. On the other hand, it is costly to hide her private signal if it is strong. This trade-off will reflect in the relative values of $\Pi_{2,x}^-$ and $\Pi_{2,x}^+$ which are defined below. Hereafter, I set $\sigma_u = 1$ for convenience. By direct integration,

one can derive their expressions:

$$\begin{aligned}
\Pi_{2,x}^-(v, x_1; K) &\equiv \mathbb{E}[\Pi_{2,x} \mathbf{1}_{|\tilde{y}_1| < K} | \mathcal{I}_{1,x}] = \mathbb{E} \left[\frac{(v - \lambda_1 \tilde{y}_1)^2}{4\lambda_2} \mathbf{1}_{|\tilde{y}_1| < K} \middle| \mathcal{I}_{1,x} \right] \\
&= \frac{(1 - \mu)[2v - \lambda_1(K + x_1)]\phi(K - x_1)}{4} - \frac{(1 - \mu)[2v + \lambda_1(K - x_1)]\phi(K + x_1)}{4} \\
&\quad + \frac{(1 - \mu)[(v - \lambda_1 x_1)^2 + \lambda_1^2]}{8\lambda_1} \left[\operatorname{erf} \left(\frac{K - x_1}{\sqrt{2}} \right) + \operatorname{erf} \left(\frac{K + x_1}{\sqrt{2}} \right) \right], \tag{124}
\end{aligned}$$

$$\begin{aligned}
\Pi_{2,x}^+(v, x_1; K) &\equiv \mathbb{E}[\Pi_{2,x} \mathbf{1}_{|\tilde{y}_1| > K} | \mathcal{I}_{1,x}] \\
&= \mathbb{E} \left[\frac{[(N + 2)v - N\hat{v}_T(\tilde{y}_1) - 2\lambda_1 \tilde{y}_1]^2}{4(N + 2)^2 \lambda_2} \mathbf{1}_{|\tilde{y}_1| > K} \middle| \mathcal{I}_{1,x} \right] \\
&= \frac{(1 - \mu)(N\rho + 2)}{4(N + 2)^2} [-2Nw\rho\lambda_1 - 2(N + 2)v + \lambda_1(N\rho + 2)(K + x_1)]\phi(K - x_1) \\
&\quad + \frac{(1 - \mu)(N\rho + 2)}{4(N + 2)^2} [-2Nw\rho\lambda_1 + 2(N + 2)v + \lambda_1(N\rho + 2)(K - x_1)]\phi(K + x_1) \\
&\quad + \frac{1 - \mu}{8(N + 2)^2 \lambda_1} \{ [(N + 2)v + Nw\rho\lambda_1 - (N\rho + 2)\lambda_1 x_1]^2 + \lambda_1^2 (N\rho + 2)^2 \} \operatorname{erfc} \left(\frac{K - x_1}{\sqrt{2}} \right) \\
&\quad + \frac{1 - \mu}{8(N + 2)^2 \lambda_1} \{ [(N + 2)v - Nw\rho\lambda_1 - (N\rho + 2)\lambda_1 x_1]^2 + \lambda_1^2 (N\rho + 2)^2 \} \operatorname{erfc} \left(\frac{K + x_1}{\sqrt{2}} \right). \tag{125}
\end{aligned}$$

where $w = (1 + c)\kappa\sigma_u$ is the horizontal intercept of $\hat{v}_T(y_1)$ and where $\phi(K \pm x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(K \pm x)^2}{2}}$ denotes the probability density function of the standard normal distribution (with $\sigma_u = 1$). Taking the first derivative, $\frac{d\Pi_x}{dx_1} = 0$, one can find the FOC for $X_1(v; K) = x_1$:

$$\begin{aligned}
0 &= v - 2\lambda_1 x_1 - \frac{(1 - \mu)(v - \lambda_1 x_1)}{4} \left[\operatorname{erf} \left(\frac{K - x_1}{\sqrt{2}} \right) + \operatorname{erf} \left(\frac{K + x_1}{\sqrt{2}} \right) \right] \\
&\quad + \frac{(1 - \mu)[(v - \lambda_1 K)^2 + 2\lambda_1^2]}{4\lambda_1} [\phi(K + x_1) - \phi(K - x_1)] + (1 - \mu)Kv\phi(K + x_1) \\
&\quad + \frac{(1 - \mu)\phi(K - x_1)}{4\lambda_1(N + 2)^2} \{ [K\lambda_1(N\rho + 2) - (N + 2)v]^2 + 2\lambda_1^2(N\rho + 2)^2 \\
&\quad \quad + \lambda_1 w N\rho [\lambda_1 w N\rho - 2K\lambda_1(N\rho + 2) + 2(N + 2)v] \} \\
&\quad - \frac{(1 - \mu)\phi(K + x_1)}{4\lambda_1(N + 2)^2} \{ [K\lambda_1(N\rho + 2) + (N + 2)v]^2 + 2\lambda_1^2(N\rho + 2)^2 \\
&\quad \quad + \lambda_1 w N\rho [\lambda_1 w N\rho - 2K\lambda_1(N\rho + 2) - 2(N + 2)v] \} \\
&\quad - \frac{(1 - \mu)(N\rho + 2)}{4(N + 2)^2} [(N + 2)v + Nw\rho\lambda_1 - (N\rho + 2)\lambda_1 x_1] \operatorname{erfc} \left(\frac{K - x_1}{\sqrt{2}} \right) \\
&\quad - \frac{(1 - \mu)(N\rho + 2)}{4(N + 2)^2} [(N + 2)v - Nw\rho\lambda_1 - (N\rho + 2)\lambda_1 x_1] \operatorname{erfc} \left(\frac{K + x_1}{\sqrt{2}} \right). \tag{126}
\end{aligned}$$

This FOC equation defines the informed trader's optimal strategy $X_1 = x_1(v; K)$ at $t = 1$.

The unconditional expected total profit of all arbitrageurs is

$$\Pi_z^{tot} \equiv \mathbb{E} \left[\sum_{n=1}^N \tilde{\pi}_{z,n}(\tilde{v}, \tilde{u}_1, \tilde{u}_2) \right] = \mathbb{E}[(\tilde{v} - \tilde{p}_2)Z_2 \mathbf{1}_{|y_1| > K}]. \quad (127)$$

After solving $x_1 = X_1(v; K)$ given any v , one can compute the conditional expected profit:

$$\begin{aligned} & \mathbb{E} \left[\sum_{n=1}^N \tilde{\pi}_{z,n}(\tilde{v}, \tilde{u}_1, \tilde{u}_2) \middle| \tilde{v} = v \right] \\ = & \mathbb{E} \left[(\tilde{v} - \lambda_1 \tilde{y}_1 - \lambda_2 (X_2(\tilde{v}, \tilde{y}_1) + Z_2(\tilde{y}_1))) Z_2(\tilde{y}_1) \mathbf{1}_{|\tilde{y}_1| > K} \middle| \tilde{v} = v \right] \\ = & \frac{N(1-\mu)}{2(N+2)^2} [w\lambda_1\rho(2N\rho+2-N) + (\rho-1)((N+2)v - \lambda_1(N\rho+2)(x_1+K))] \phi(K-x_1) \\ & + \frac{N(1-\mu)}{2(N+2)^2} [w\lambda_1\rho(2N\rho+2-N) - (\rho-1)((N+2)v - \lambda_1(N\rho+2)(x_1-K))] \phi(K+x_1) \\ & - \frac{N(1-\mu)}{4(N+2)^2} [(N+2)v(\rho(w-x_1)+x_1) + \lambda_1 N\rho^2(1+(w-x_1)^2) \\ & \quad - \lambda_1\rho(N-2)(x_1^2 - wx_1 + 1) - 2\lambda_1(1+x_1^2)] \operatorname{erfc} \left(\frac{K-x_1}{\sqrt{2}} \right) \\ & + \frac{N(1-\mu)}{4(N+2)^2} [(N+2)v(\rho(w+x_1)-x_1) - \lambda_1 N\rho^2(1+(w+x_1)^2) \\ & \quad + \lambda_1\rho(N-2)(x_1^2 + wx_1 + 1) + 2\lambda_1(1+x_1^2)] \operatorname{erfc} \left(\frac{K+x_1}{\sqrt{2}} \right), \end{aligned} \quad (128)$$

where $w \equiv (1+c)\kappa\sigma_u$ and $\sigma_u = 1$. Finally, the unconditional total payoff to arbitrageurs is

$$\Pi_z^{tot} = \mathbb{E} \left[\sum_{n=1}^N \tilde{\pi}_{z,n}(\tilde{v}, \tilde{u}_1, \tilde{u}_2) \right] = \int_{-\infty}^{+\infty} f_L(v) \mathbb{E} \left[\sum_{n=1}^N \tilde{\pi}_{z,n}(\tilde{v}, \tilde{u}_1, \tilde{u}_2) \middle| \tilde{v} = v \right] dv. \quad (129)$$

A.11 Proof of Corollary 3.3

Since $\lim_{\mu \rightarrow 0} c = -1$ and $\lim_{\mu \rightarrow 0} \rho = 2 + \frac{2}{N}$, one can derive that for the informed trader

$$\begin{aligned} \lim_{v \rightarrow 0} \lim_{\mu \rightarrow 0} \Pi_x &= \lambda_1 - \frac{3\lambda(1+x_1^2)}{8} \left[\operatorname{erf} \left(\frac{K-x_1}{\sqrt{2}} \right) + \operatorname{erf} \left(\frac{K+x_1}{\sqrt{2}} \right) \right] \\ &+ \frac{3\lambda_1}{4} \{ [\phi(K-x_1) + \phi(K+x_1)] K + [\phi(K-x_1) - \phi(K+x_1)] x_1 \} \end{aligned} \quad (130)$$

which only depends on x_1 , λ_1 , and K . The FOC equation in this limiting case becomes

$$\frac{3\lambda}{4} \left[(2 + K^2) [\phi(K - x_1) - \phi(K + x_1)] - x_1 \left(\operatorname{erf} \left(\frac{K - x_1}{\sqrt{2}} \right) + \operatorname{erf} \left(\frac{K + x_1}{\sqrt{2}} \right) \right) \right] = 0. \quad (131)$$

Using the equilibrium threshold $K^*(\mu = 0) = \kappa$ with $\sigma_u = 1$, we can rewrite the FOC as:

$$\frac{x_1}{2 + \kappa^2} = \frac{\phi(\kappa - x_1) - \phi(\kappa + x_1)}{\operatorname{erf} \left(\frac{\kappa - x_1}{\sqrt{2}} \right) + \operatorname{erf} \left(\frac{\kappa + x_1}{\sqrt{2}} \right)}, \quad (132)$$

which may have multiple solutions: one is obviously $x_1 = 0$ and the other two are $\pm\infty$.

As long as the informed trader trades a sufficiently large quantity $x_1 \gg \kappa$ (instead of $\pm\infty$), the probability of triggering arbitrageurs to trade is arbitrarily close to one. In the second period, the informed trader's optimal strategy is found to be $\lim_{\mu \rightarrow 0} X_2(v = 0, y_1) = -y_1$, which exactly offsets the total quantity traded by arbitrageurs $\lim_{\mu \rightarrow 0} Z_2(y_1) = y_1$. Thus, the terminal position of the informed trader is $x_1 + x_2 = -u_1$ which is zero on average. The expected profit from this disruptive strategy is found to be $\Pi_x \approx \lambda_1 \sigma_u^2$, which is limited by the noise trading volatility in the first period.